CHAPTER

15

WAVES

15.1 WAVE MOTION

1. *Define the term wave motion. Explain it with the help of a suitable example. Give important characteristics of wave motion.*

Wave motion. *Wave motion is a kind of disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next.* In a wave, both information and energy propagate (in the form of signals) from one point to another but there is no motion of matter as a whole through a medium.

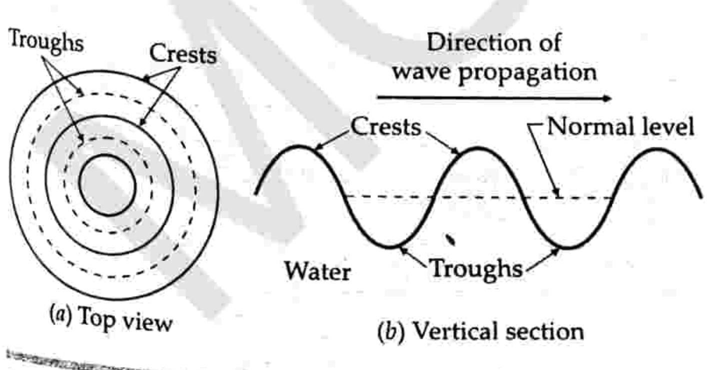


Fig. 15.1 Formation of waves over a water surface.

If we drop a pebble into a pond of still water, a circular pattern of alternate crests and troughs spreads out from the point where the pebble strikes the water surface. The kinetic energy of the pebble makes the particles oscillate which come in contact with it. These particles, in turn, transfer energy to the particles of next layer which also begin to oscillate. Energy is further transferred to the particles of next layer which also begin to oscillate and so on. In this way energy is transferred from one point to another. Further, if we throw a piece of paper or a cork on the water surface, it is found to oscillate up and down about the mean position and does not move forward with the wave. This shows that it is the disturbance or the wave which travels forward and not the particles of the medium.

Characteristics of wave motion. Some of the important characteristics of wave motion are as follows :

(*i*) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(*ii*) The energy is transferred from one place to another without any actual transfer of the particles of the medium.

(*iii*) Each particle receives disturbance a little later than its preceding particle *i.e.*, there is a regular phase difference between one particle and the next.

(*iv*) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.

(*v*) The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.

(*vi*) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

2. *What are the different types of waves we come across ? Give examples of each type.*

Different types of waves. The waves we come across are mainly of *three* types :

(*i*) **Mechanical waves.** *The waves which require a material medium for their propagation are called mechanical waves.* Such waves are also called elastic waves because their propagation depends on the elastic properties of the medium. These waves are governed by Newton's laws and can exist in a material mediums, such as water, air, rock etc.

Examples. Water waves, sound waves, seismic waves (waves produced during earthquake), etc.

(*ii*) **Electromagnetic waves.** *The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation and are also called non-mechanical waves.* Light from the sun and distant stars reaches us through inter-stellar space, which is almost vacuum. All electromagnetic waves travel through vacuum at the same speed *c*, given by

$$c = 29,97,92,458 \text{ ms}^{-1} \quad \text{(speed of light)}$$

Examples. Visible and ultraviolet light, radiowaves, microwaves, X-rays, etc.

(*iii*) **Matter waves.** *The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves.* These waves become important in the quantum mechanical description of matter.

Examples. Electron microscopes make use of the matter waves associated with fast moving electrons.

15.2 SPRING-MODEL FOR PROPAGATION OF A WAVE THROUGH AN ELASTIC MEDIUM

3. *How the propagation of a wave through an elastic medium can be explained on the basis of spring-model. Hence explain the propagation of a sound wave in (a) air and (b) solids.*

Spring-model for the propagation of a wave through an elastic medium. As shown in Fig. 15.2, consider a number of springs connected to one another. One end is fixed to a rigid support. The first spring is pulled to the left and released. It gets stretched. Due to elasticity, a restoring force is developed in the first spring. This force brings to the first spring back to its original size and stretches the second spring, and so on. Thus the disturbance moves from one end to the other, but each spring only executes small oscillations about its equilibrium position or length.

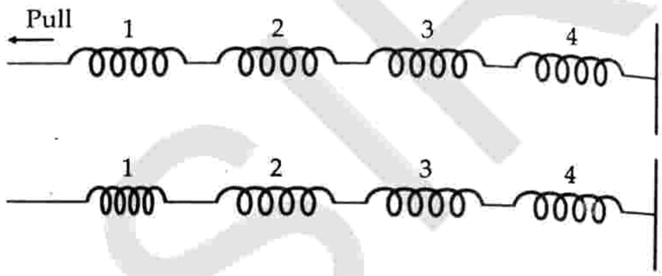


Fig. 15.2 Propagation of a disturbance through a combination of springs.

The above spring model can be used to explain the propagation of sound waves through air or any solid.

(*a*) **Propagation of sound waves through air.** A small region of air can be considered as a spring. It is connected to the neighbouring regions or springs. As sound wave travels through air, it compresses or expands a small region of air. This changes the density and pressure of the region.

According to Boyle's law,

Change in pressure $(\Delta P) \propto$ change in density $(\Delta \rho)$

As the pressure is force per unit area, so a restoring force proportional to the change in density is developed, just like in an extended or compressed spring. If a region is compressed, its molecules tend to move out to the adjacent region, thereby increasing the density or creating *compression* in that region. The density of air in the first region decreases and is called *rarefaction*. But when a region is rarefied, the air of the surrounding air rushes in. This shifts the rarefaction to the adjacent region. Thus compressions and rarefactions move from one region to another. This makes possible the propagation of disturbance in air.

(*b*) **Propagation of sound in a solid.** In a crystalline solid, various atoms can be considered as end points, with springs connected between pairs of them. Each atom is in its state of equilibrium, as the forces exerted by the other atoms are cancelled out. When an elastic (sound) wave propagates, the atom is displaced from its equilibrium position and a restoring force is developed. The disturbance produced by the force travels to the next atom and so on. Thus the wave propagates through the solid.

15.3 TRANSVERSE AND LONGITUDINAL WAVES

4. *What are transverse and longitudinal waves ? Explain with suitable examples.*

Types of wave motion. Depending on the relationship between the direction of oscillation of individual particles and the direction of wave propagation, the waves are classified into *two* categories : transverse waves and longitudinal waves.

Transverse waves. *These are the waves in which the individual particles of the medium oscillate perpendicular to the direction of wave propagation.* As shown in Fig. 15.3(*a*), consider a horizontal string with its one end fixed to a rigid support and other end held in the hand. If we give its free end a smart upward jerk, an upward kink or pulse is created there which travels along the string towards the fixed end. Each part of the string successively undergoes a disturbance about its mean position. As shown in Fig. 15.3(*b*), if we continuously give up and down jerks to the free end of the string, a number of sinusoidal waves begin to travel along the string.

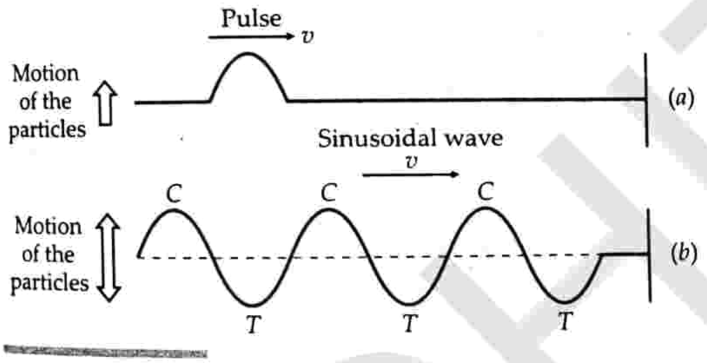


Fig. 15.3 (*a*) A single pulse, (*b*) A sinusoidal wave sent along a stretched string.

Each part of the string vibrates up and down while the wave travels along the string. So the waves in the string are transverse in nature.

The points ($C, C, \ldots$) of maximum displacement in the upward direction are called *crests*. The points ($T, T, \ldots$) of maximum displacement in the downward direction are called *troughs*. *One crest and one trough together form one wave.*

Longitudinal waves. *These are the waves in which the individual particles of the medium oscillate along the direction of wave propagation.*

As shown in Fig. 15.4, consider a long hollow cylinder *AB* closed at one end and having a movable piston at the other end. If we suddenly move the piston rapidly towards right, a small layer of air just near the piston-head is compressed and after being compressed, this layer moves towards right and compresses the next layer and soon the compression reaches the other end. Now if the piston is suddenly moves towards left, the layer adjacent to it is rarefied resulting in the fall of pressure. The air from the next layer moves in to restore pressure. Consequently the next layer is rarefied. In this way a pulse of rarefaction moves towards right.

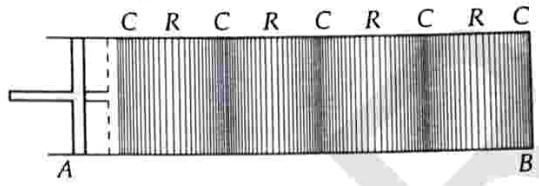


Fig. 15.4 A sound wave produced in a cylinder by moving a piston back and forth.

If we continuously push and pull the piston in a simple harmonic manner, a sinusoidal sound wave travels along the cylinder in the form of alternate compressions and rarefactions, marked *C, R, C, R*, etc. As the oscillations of an element of air are parallel to direction of wave propagation, the wave is a longitudinal wave. Hence sound waves produced in air are longitudinal waves.

5. *Mention the important properties which a medium must possess for the propagation of mechanical waves through it.*

Essential properties of a medium for the propagation of mechanical waves. Both transverse and longitudinal waves can propagate through those media which have the following properties :

(*i*) ***Elasticity.*** The medium must possess elasticity so that the particles can return to their mean positions after being disturbed.

(*ii*) ***Inertia.*** The medium must possess inertia or mass so that its particles can store kinetic energy.

(*iii*) ***Minimum friction.*** The frictional force amongst the particles of the medium should be negligibly small so that they continue oscillating for a sufficiently long time and the wave travels a sufficiently long distance through the medium.

6. *Through what type of media, can (i) transverse waves and (ii) longitudinal waves be transmitted ? Give reason.*

(*i*) **Media through which transverse waves can propagate.** Transverse waves travel in the form of crests and troughs. They involve changes in the shape of the medium. So they can be transmitted through media which have rigidity. As solids and strings can sustain shearing stress, so transverse waves can be formed in solids and strings, not in fluids.

Due to surface tension, the free surface of liquid tends to maintain its level. So transverse waves can be formed over liquid surfaces.

(*ii*) **Media through which longitudinal waves can propagate.** Longitudinal waves travel in the form of compressions and rarefactions. They involve changes in volume and density of the medium. All media—solids, liquids and gases can sustain compressive stress, so longitudinal waves can be transmitted through all the three types of media.

15.4 SOME DEFINITIONS IN CONNECTION WITH WAVE MOTION

7. *In reference to a wave motion, define the terms (i) amplitude, (ii) time period, (iii) frequency, (iv) angular frequency, (v) wavelength, (vi) wave number, (vii) angular wave number and (viii) wave velocity.*

Some definitions in connection with wave motion. When a transverse or a longitudinal wave propagates through a medium, all the particles of the medium oscillate about the mean positions in the same manner but the phase of oscillation changes from one particle to the next.

(*i*) **Amplitude.** *It is the maximum displacement suffered by the particles of the medium about their mean positions.* It is denoted by A.

(*ii*) **Time period.** *The time period of a wave is the time in which a particle of medium completes one vibration to and fro about its mean position.* It is denoted by T.

(*iii*) **Frequency.** *The frequency of a wave is the number of waves produced per unit time in the given medium.* It is equal to the number of oscillations completed per unit time by any particle of the medium. It is equal to the reciprocal of the time period T of the particle and is denoted by ν. Thus

$$\nu = \frac{1}{T}$$

SI unit of ν is s^{-1} or hertz (Hz).

(*iv*) **Angular frequency.** *The rate of change of phase with time is called angular frequency of the wave.* It is clearly equal to $2\pi/T$, because the phase change in time T is 2π. It is denoted by ω. Thus

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

SI unit of $\omega = \text{rad s}^{-1}$.

(*v*) **Wavelength.** *It is the distance covered by a wave during the time in which a particle of the medium completes one vibration to and fro about its mean position. Or, it is the distance between two nearest particles of the medium which are vibrating in the same phase.* It is denoted by λ.

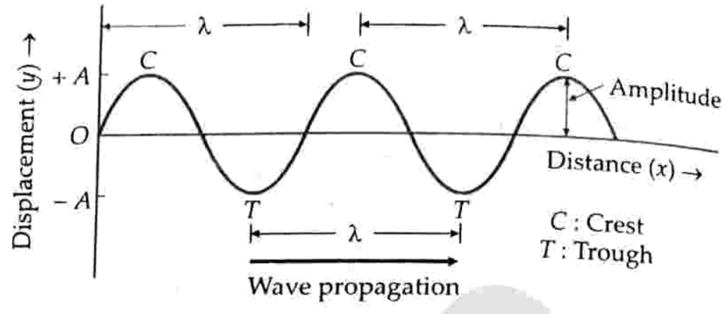


Fig. 15.5 Displacement-distance (y–x) graph for a transverse wave.

In a transverse wave, the distance between two successive crests or troughs is equal to the wavelength λ, as shown in Fig. 15.5. In a longitudinal wave, the distance between the centres of two nearest compressions or rarefactions is equal to wavelength λ.

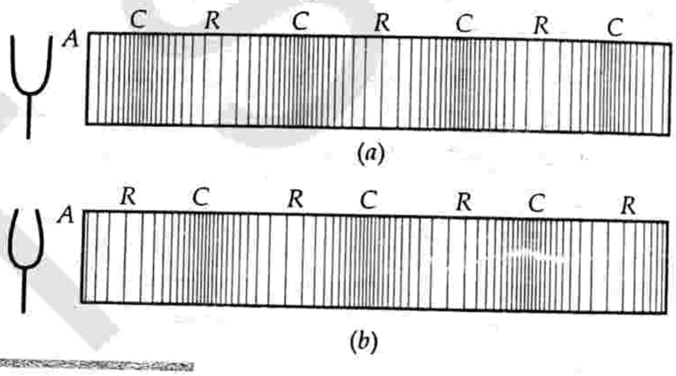


Fig. 15.6 Sound (longitudinal) waves from a tuning fork.

(*vi*) **Wave number.** *The number of waves present in a unit distance of the medium is called wave number.* It is equal to the reciprocal of wavelength λ. Thus

$$\text{Wave number, } \bar{\nu} = \frac{1}{\lambda}$$

SI unit of wave number = m^{-1}

(*vii*) **Angular wave number or propagation constant.** *The quantity $2\pi/\lambda$ is called angular wave number or propagation constant of a wave.* It represents the phase change per unit path difference. It is denoted by k. Thus

$$k = \frac{2\pi}{\lambda}$$

The SI unit of k is radian per metre or rad m^{-1}.

(*viii*) **Wave velocity or phase velocity.** *The distance covered by a wave per unit time in its direction of propagation is called its wave velocity or phase velocity.* It is denoted by v.

8. *Derive relation between wave velocity, frequency and wavelength of a wave.*

Relation between wave velocity, frequency and wavelength. We know that when a particle of the medium completes one oscillation about its mean position in periodic time T, the wave travels a distance equal to its wavelength λ. Therefore,

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}}$$

or $$v = \frac{\lambda}{T}$$

or $$v = \nu\lambda \qquad [\because \nu = 1/T]$$

i.e., Wave velocity = Frequency × Wavelength

Examples based on Relation between Frequency, Wavelength and Wave Velocity

FORMULAE USED

1. Wave velocity = Frequency × Wavelength
   or $v = \nu\lambda$
2. $\text{Wave velocity} = \dfrac{\text{Wavelength}}{\text{Time period}}$ or $v = \dfrac{\lambda}{T}$
3. $\text{Wavelength} = \dfrac{\text{Wave velocity}}{\text{Frequency}}$ or $\lambda = \dfrac{v}{\nu}$

UNITS USED

Wavelength λ is in metre, frequency ν in Hz or s^{-1}, time period T in second and wave velocity v in ms^{-1}.

EXAMPLE 1. *How far does the sound travel in air when a tuning fork of frequency 256 Hz makes 64 vibrations ? Velocity of sound in air = 320 ms^{-1}.* **[Delhi 05]**

Solution. Here $\nu = 256$ Hz,

$$v = 320 \text{ ms}^{-1}$$

Distance travelled by the wave in one vibration is equal to its wavelength.

$$\therefore \quad \lambda = \frac{v}{\nu} = \frac{320}{256} = 125 \text{ m}$$

Distance travelled by the wave in 64 vibrations

$= 125 \times 64 =$ **80 m.**

EXAMPLE 2. *A source of sound is placed at one end of an iron bar two kilometre long and two sounds are heard at the other end at an interval of 5.6 seconds. If the velocity of sound in air is 330 ms^{-1}, find the velocity of sound in iron.*

Solution. One sound is heard through air and another through iron.

Time taken by sound in air,

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{2000 \text{ m}}{330 \text{ ms}^{-1}} = 6.06 \text{ s}$$

As the interval between the two sounds is 5.6 s and sound travels faster in iron than in air, so the time taken by sound in iron is

$$t' = 6.06 - 5.6 = 0.46 \text{ s}$$

Velocity of sound in iron

$$= \frac{\text{Distance}}{\text{Time}} = \frac{2000 \text{ m}}{0.46 \text{ s}} = \mathbf{4348 \text{ ms}^{-1}}.$$

EXAMPLE 3. *Audible frequencies have a range 20 Hz to 20,000 Hz. Express this range in terms of (i) period T (ii) wavelength λ in air and (iii) angular frequency. Given velocity of sound in air is 330 ms^{-1}.*

Solution. Here $\nu_1 = 20$ Hz, $\nu_2 = 20{,}000$ Hz,

$$v = 330 \text{ ms}^{-1}$$

(*i*) $$T_1 = \frac{1}{\nu_1} = \frac{1}{20} = 5 \times 10^{-2} \text{ s},$$

$$T_2 = \frac{1}{\nu_2} = \frac{1}{20{,}000} = 5 \times 10^{-5} \text{ s}$$

Thus the audible range in terms of period is from 5×10^{-2} s to 5×10^{-5} s.

(*ii*) $$\lambda_1 = \frac{v}{\nu_1} = \frac{330}{20} = 16.5 \text{ m}$$

$$\lambda_2 = \frac{v}{\nu_2} = \frac{330}{20{,}000} = 0.0165 \text{ m}$$

Thus the audible range in terms of wavelength is from 16.5 m to 0.0165 m.

(*iii*) $$\omega_1 = 2\pi\nu_1 = 2\pi \times 20 = 40\pi \text{ rad s}^{-1}$$

$$\omega_2 = 2\pi\nu_2 = 2\pi \times 20{,}000 = 40{,}000\pi \text{ rad s}^{-1}$$

Thus the audible range in terms of angular frequency is from 4π rad s^{-1} to 40,000 π rad s^{-1}.

PROBLEMS FOR PRACTICE

1. A radio station broadcasts its programme at 219.3 metre wavelength. Determine the frequency of radio waves if velocity of radio waves be 3×10^8 ms^{-1}. (**Ans.** 1.368×10^6 Hz)
2. The audible frequency range of a human's ear is 20 Hz – 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperatures to be 340 ms^{-1}. (**Ans.** 0.017 m to 17 m)
3. The speed of a wave in a medium is 960 ms^{-1}. If 3600 waves are passing through a point in the medium in 1 minute, then calculate the wavelength. (**Ans.** 16 m)
4. If a splash is heard 4.23 seconds after a stone is dropped into a well 78.4 m deep, find the speed of sound in air. (**Ans.** 340.87 ms^{-1})

5. A stone is dropped into a well and its splash is heard at the mouth of the well after an interval of 1.45 s. Find the depth of the well. Given that velocity of sound in air at room temperature is equal to 332 ms^{-1}. **[MNREC 81]**

(Ans. 9.9 m)

6. A body sends waves 100 mm long through medium A and 0.25 m long in medium B. If the velocity of waves in medium A is 80 cm s^{-1}, calculate the velocity of waves in medium B. **(Ans.** 2 ms^{-1})

HINTS

2. Here $\lambda_1 = \frac{v}{\nu_1} = \frac{340}{20} = \mathbf{17\ m.}$

and $\lambda_2 = \frac{v}{\nu_2} = \frac{340}{20 \times 10^3} = \mathbf{0.017\ m.}$

3. Speed of the wave,

$$v = 960\ ms^{-1}$$

Frequency of the wave,

$$\nu = 3600\ min^{-1} = \frac{3600}{60} = 60\ s^{-1}$$

Wavelength,

$$\lambda = \frac{v}{\nu} = \frac{960}{60} = \mathbf{16\ m.}$$

4. For downward motion of the stone,

$u = 0$, $a = 9.8\ ms^{-2}$, $s = 74.8$ m, $t = ?$

As $s = ut + \frac{1}{2}at^2$

$\therefore$ $78.4 = 0 + \frac{1}{2} \times 9.8\, t^2 = 4.9\, t^2$

or $t^2 = \frac{78.4}{4.9} = 16$ or $t = 4$ s

Let t' be the time taken by the splash of sound to reach the top of the well. Then

$$t + t' = 4 + t' = 4.23\ s \quad \text{or} \quad t' = 4.23 - 4 = 0.23\ s$$

Speed of sound in air

$$= \frac{\text{Distance}}{\text{Time}} = \frac{78.4}{0.23} = \mathbf{340.87\ ms^{-1}.}$$

5. Let h be the depth of the well. Then time t_1 taken by the stone to fall into well under gravity is given by

$$h = 0 + \frac{1}{2}gt_1^2 \quad \text{or} \quad t_1 = \sqrt{\frac{2h}{g}}$$

Time taken for the splash to travel height h is given by

$$t_2 = \frac{h}{v}$$

where v = velocity of sound

But $t_1 + t_2 = 1.45$ s

$\therefore$ $\sqrt{\frac{2h}{g}} + \frac{h}{v} = 1.45$

or $\sqrt{\frac{2h}{9.8}} + \frac{h}{332} = 1.45$

On solving, $h = \mathbf{9.9\ m.}$

6. Here $\lambda_A = 100$ mm $= 0.10$ m, $\lambda_B = 0.25$ m,

$v_A = 80$ cm $s^{-1} = 0.80\ ms^{-1}$

As the frequency of the wave remains same in the two media, so

$$\nu = \frac{v_A}{\lambda_A} = \frac{v_B}{\lambda_B}$$

$$\therefore \quad v_B = \frac{\lambda_B}{\lambda_A} \times v_A = \frac{0.25}{0.10} \times 0.80 = \mathbf{2\ ms^{-1}.}$$

15.5 SPEED OF TRANSVERSE WAVES

9. *On the basis of dimensional considerations, write the formula for the speed of transverse waves (a) on a stretched string and (b) in a solid.*

(*a*) **Speed of a transverse wave on a stretched string.** The wave velocity through a medium depends on its inertial and elastic properties. So the speed of transverse wave through a stretched string is determined by two factors :

(*i*) Tension T in the string is a measure of elasticity in the string. Without tension no disturbance can propagate in the string.

Dimensions of T = [Force] = $[MLT^{-2}]$

(*ii*) Mass per unit length or *linear mass density* m of the string so that the string can store kinetic energy.

$$\text{Dimensions of } m = \frac{[\text{Mass}]}{[\text{Length}]} = [ML^{-1}]$$

Now, dimensions of ratio $\frac{T}{m} = \frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}]$

As the speed v has the dimensions $[LT^{-1}]$, so we can express v in terms of T and m as

$$v = C\sqrt{\frac{T}{m}}$$

From detailed mathematical analysis or from experiments, the dimensionless constant $C = 1$. Hence the speed of transverse waves on a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

Clearly, *the speed of a transverse wave along a stretched string depends only on the tension T and linear mass density m of the string. It does not depend on the frequency of the wave.* The frequency of a wave depends on the source generating that wave.

(*b*) **Speed of transverse wave in a solid.** The speed of transverse wave through a solid is determined by *two* factors : (*i*) Elasticity of shape or modulus of rigidity η of the solid. (*ii*) Mass per unit volume or density ρ determines its inertia.

Now,

Dimensions of ratio $\frac{\eta}{\rho} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$

Dimensions of speed $v = [LT^{-1}]$

So we can express v in terms of η and ρ as

$$v = C\sqrt{\frac{\eta}{\rho}}$$

The dimensionless constant C is found to be unity. Thus the speed of transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}.$$

Examples based on Velocity of Transverse Waves in Solids and Strings

Formulae Used

1. Velocity of transverse waves in a solid of modulus of rigidity η and density ρ,
$$v = \sqrt{\frac{\eta}{\rho}}$$
2. Velocity of transverse waves in a string of mass per unit length m and stretched under tension T,
$$v = \sqrt{\frac{T}{m}}$$

Units Used

Here η is Nm^{-2}, ρ in $kg\ m^{-3}$, tension T in N, linear mass density in $kg\ m^{-1}$ and velocity v in ms^{-1}.

Example 4. *For aluminium the modulus of rigidity is* $2.1 \times 10^{10}\ Nm^{-2}$ *and density is* $2.7 \times 10^3\ kg\ m^{-3}$. *Find the speed of transverse waves in the medium.*

Solution. Here $\eta = 2.1 \times 10^{10}\ Nm^{-2}$,

$$\rho = 2.7 \times 10^3\ kg\ m^{-3}$$

Speed of transverse waves in aluminium is given by

$$v = \sqrt{\frac{\eta}{\rho}} = \sqrt{\frac{2.1 \times 10^{10}}{2.7 \times 10^3}} = \mathbf{2.79 \times 10^3\ ms^{-1}}.$$

Example 5. *A steel wire 0.72 m long has a mass of* 5.0×10^{-3} *kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire ?* **[NCERT]**

Solution. Here $T = 60$ N, Mass $= 5.0 \times 10^{-3}$ kg, Length $= 0.72$ m

Mass per unit length,

$$m = \frac{5.0 \times 10^{-3}\ kg}{0.72\ m} = \mathbf{6.9 \times 10^{-3}\ kg\ m^{-1}}$$

The speed of the transverse wave on the wire,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{60\ N}{6.9 \times 10^{-3} kgm^{-1}}} = \mathbf{93\ ms^{-1}}.$$

Example 6. *In a sonometer experiment, the density of the material of the wire used is* $7.5 \times 10^3\ kg\ m^{-3}$. *If the stress of the wire is* $3.0 \times 10^8\ Nm^{-2}$, *find out the speed of the transverse wave in the wire.*

Solution. Let A be the area of cross-section of the wire.

Tension in the wire,

$$T = \text{Stress} \times \text{area} = 3.0 \times 10^8 \times A \text{ newton}$$

Mass per unit length,

$$m = A \times 1 \times \rho = A \times 7.5 \times 10^3\ kg\ m^{-1}$$

Speed,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{3.0 \times 10^8 \times A}{A \times 7.5 \times 10^3}} = \mathbf{200\ ms^{-1}}.$$

Example 7. *A copper wire is held at the two ends by rigid supports. At* $30°C$, *the wire is just taut with negligible tension. Find the speed of transverse waves in the wire at* $10°C$. ($\alpha = 1.7 \times 10^{-5}\ °C^{-1}$, $Y = 1.4 \times 10^{11}\ Nm^{-2}$ *and* $\rho = 9 \times 10^3\ kg\ m^{-3}$).

Solution. When the temperature changes from 30°C to 10°C, then change in length of the wire is

$$\Delta l = \alpha\, l\, \Delta T = 1.7 \times 10^{-5} \times l \times (30 - 10)$$
$$= l \times 3.4 \times 10^{-4}\ m$$

Young's modulus,

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

So the tension produced in the wire is

$$T = F = \frac{Y A \Delta l}{l} = \frac{1.4 \times 10^{11} \times A \times l \times 3.4 \times 10^{-4}}{l}$$
$$= 4.76 \times 10^7 \times A \text{ newton}$$

Mass per unit length,

$$m = A \times 1 \times \rho = A\rho = A \times 9 \times 10^3\ kg\ m^{-1}$$

Speed of transverse wave,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{4.76 \times 10^7 \times A}{A \times 9 \times 10^3}} = \mathbf{72\ ms^{-1}}.$$

PROBLEMS FOR PRACTICE

1. A steel wire 70 cm long has a mass of 7 kg. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire ? (**Ans.** 100 ms^{-1})

2. The speed of a transverse wave in a stretched string is 348 ms^{-1}, when the tension of the string is 3.6 kg wt. Calculate the speed of the transverse wave in the same string, if the tension in the string is changed to 4.9 kg wt. (**Ans.** 406 ms^{-1})

3. Calculate the velocity of transverse waves in a copper wire 1 mm^2 in cross-section, under the tension produced by 1 kg wt. The density of copper = 8.93 kg m^{-3}. (**Ans.** 33.12 ms^{-1})

4. A wave-pulse is travelling on a string of linear mass density 10 g cm^{-1} under a tension of 1 kg wt. Calculate the time taken by the pulse to travel a distance of 50 cm on the string. Given $g = 10\ ms^{-2}$. (**Ans.** 0.05 s)

5. The diameter of an iron wire is 1.20 mm. If the speed of the transverse wave in the wire be 50.0 ms^{-1}, what is the tension in the wire ? The density of iron is 7.7×10^3 kg m^{-3}. (**Ans.** 21.78 N)

HINTS

2. $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \qquad \text{(For a given string)}$$

$$\therefore \quad v_2 = \sqrt{\frac{T_2}{T_1}} \times v_1 = \sqrt{\frac{4.9 \times g}{3.6 \times g}} \times 348$$

$$= \frac{7}{6} \times 348 = \mathbf{406\ ms^{-1}}.$$

3. Here $A = 1\ mm^2 = 10^{-6}\ m^2$, $\rho = 8.93 \times 10^3$ kg m^{-3},

$$T = 1 \text{ kg wt} = 9.8 \text{ N}$$

Mass per unit length of wire,

$$m = A \times 1 \times \rho = 10^{-6} \times 8.93 \times 10^3$$

$$= 8.93 \times 10^{-3} \text{ kg m}^{-1}$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{9.8}{8.93 \times 10^{-3}}}$$

$$= \mathbf{33.12\ ms^{-1}}.$$

4. Here $$m = \frac{1\text{g}}{1\text{cm}} = \frac{10^{-3}\text{ kg}}{10^{-2}\text{ m}} = 10^{-1} \text{ kg m}^{-1}$$

$$T = 1 \text{ kg wt} = 10 \text{ N}$$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{10}{10^{-1}}} = 10\ ms^{-1}$$

Time taken to travel 50 cm $= \dfrac{50 \times 10^{-2}\text{ m}}{10\ ms^{-1}} = \mathbf{0.05\ s.}$

5. Mass per unit length,

$$m = A \times 1 \times \rho = \frac{\pi d^2 \rho}{4}$$

As $$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

$$\therefore \quad T = mv^2 = \frac{\pi d^2 \rho v^2}{4}$$

$$= \frac{22 \times (1.20 \times 10^{-3})^2 \times 7.7 \times 10^3 (50)^2}{7 \times 4}$$

$$= \mathbf{21.78\ N.}$$

15.6 SPEED OF A LONGITUDINAL WAVE

10. *Write expression for the speed of a longitudinal wave in (a) a liquid or gas, (b) a solid and (c) a long solid rod.*

(*a*) **Speed of a longitudinal wave in a liquid or gas.** In a longitudinal wave, the particles of the medium oscillate forward and backward in the direction of propagation of the wave. They cause compressions and rarefactions of small volume elements of fluid. So the speed of a longitudinal wave through a fluid is determined by two factors :

(*i*) The volume elasticity or bulk modulus κ of the fluid.

(*ii*) The density of the fluid which determines its inertia.

$\therefore$ Dimensions of the ratio $\dfrac{\kappa}{\rho}$

$$= \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$$

Dimensions of speed $v = [LT^{-1}]$

So the speed v can be expressed in terms of κ and ρ as

$$v = C\sqrt{\frac{\kappa}{\rho}}$$

The dimensionless constant C is found to be unity. Hence the speed of a longitudinal wave in any fluid (liquid or gas) is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

Clearly, *the speed of a longitudinal wave through a fluid depends only on its bulk modulus* κ *and density* ρ.

(*b*) **Speed of a longitudinal wave in a solid.** The speed of a longitudinal wave through a solid of bulk modulus κ, modulus of rigidity η and density ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

(c) **Speed of a longitudinal wave in a solid rod.** When a long solid rod is given blows at one end, longitudinal waves travel through it in the form of compressions and rarefactions. As the sidewise expansion of the rod is negligible, we need to consider only longitudinal strain. In this case, the relevant modulus of elasticity is the Young's modulus. Hence the speed of a longitudinal wave through a solid rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}.$$

15.7 SPEED OF SOUND : NEWTON'S FORMULA AND LAPLACE CORRECTION

11. *Write Newton's formula for the speed of sound in a gas. Why and what correction was applied by Laplace in this formula ?*

Newton's formula for the speed of sound in a gas. Newton gave the first theoretical expression for the speed of sound in a gas. He assumed that sound waves travel through a gas under isothermal conditions. He argued that the small amount of heat produced in a compression is rapidly conducted to the surrounding rarefactions where slight cooling is produced. Thus the temperature of gas remains constant. If κ_{iso} is the isothermal volume elasticity (bulk modulus of the gas at constant temperature), then the speed of sound in the gas will be

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}}$$

For an isothermal change,

$$PV = \text{constant} \qquad \text{(Boyle's law)}$$

Differentiating both sides, we get

$$P\,dV + V dP = 0$$

or
$$PdV = -V\,dP$$

or
$$P = -\frac{VdP}{dV} = -\frac{dP}{dV/V}$$

$$= \frac{\text{Volume stress}}{\text{Volume strain}} = \kappa_{iso}$$

Hence the *Newton's formula* for the speed of sound in a gas is

$$v = \sqrt{\frac{P}{\rho}}$$

At STP, $P = 0.76$ m of Hg $= 0.76 \times 13.6 \times 10^3 \times 9.8$
$= 1.013 \times 10^5\ \text{Nm}^{-2}$

ρ = Density of air $= 1.293\ \text{kg m}^{-3}$.

$\therefore$ Speed of sound in air at STP,

$$v = \sqrt{\frac{1.013 \times 10^5}{1.293}} \simeq 280\ \text{ms}^{-1}.$$

This value is about 15% less than the experimental value (331 ms^{-1}) of the speed of sound in air at STP. Hence Newton's formula is not acceptable.

Laplace's correction. In 1816, the French scientist *Laplace* pointed out that sound travels through a gas under adiabatic conditions not under isothermal conditions (as suggested by Newton). This is because of the following reasons :

(i) As sound travels through a gas, temperature rises in the regions of compressions and falls in the regions of rarefactions.

(ii) A gas is a poor conductor of heat.

(iii) The compressions and rarefactions are formed so rapidly that the heat generated in the regions of compressions does not get time to pass into the regions of rarefactions so as to equalise the temperature.

So when sound travels through a gas, the temperature does not remain constant. The pressure-volume variations are adiabatic. If κ_{adia} is the adiabatic bulk modulus of the gas, then the formula for the speed of sound in the gas would be

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}}$$

For an adiabatic change, $PV^{\gamma} = \text{constant}$

Differentiating both sides, we get

$$P(\gamma V^{\gamma - 1})\,dV + V^{\gamma}\,dP = 0$$

or
$$\gamma\,PdV + VdP = 0$$

$$\gamma\,P = -\frac{dP}{dV/V} = \kappa_{adia}$$

where $\gamma = C_p / C_v$, is the ratio of two specific heats.

Hence the Laplace formula for the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

This modification of Newton's formula is known as *Laplace correction.*

For air $\gamma = 7/5$, so speed of sound in air at STP will be

$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 = 331.3\ \text{ms}^{-1}.$$

This value is in close agreement with the experimental value. Hence the Laplace correction is justified.

Examples based on Velocity of Longitudinal Waves

FORMULAE USED

1. Velocity of longitudinal waves in a solid of bulk modulus κ, modulus of rigidity η and density ρ is given by
$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$
2. Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by
$$v = \sqrt{\frac{Y}{\rho}}$$
3. Velocity of longitudinal waves in liquid of bulk modulus κ and density ρ is given by
$$v = \sqrt{\frac{\kappa}{\rho}}$$
4. *Newton's formula* for the velocity of sound in a gas is
$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$$
where P = pressure of a gas
5. *Laplace formula* for the velocity of sound in a gas is
$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}, \quad \text{where} \quad \gamma = \frac{C_p}{C_v}$$

UNITS USED

Modulii of elasticity κ, Y and η are in Nm^{-2}, pressure P in Nm^{-2} or Pa, velocity v in ms^{-1} and specific heats ratio γ has no units.

EXAMPLE 8. *For aluminium the bulk modulus and modulus of rigidity are 7.5×10^{10} Nm^{-2} and 2.1×10^{10} Nm^{-2}. Find the velocity of longitudinal waves in the medium. Density of aluminium is 2.7×10^3 $kg\ m^{-3}$.*

Solution. Here $\kappa = 7.5 \times 10^{10}\ Nm^{-2}$,

$\eta = 2.1 \times 10^{10}\ Nm^{-2}$, $\rho = 2.7 \times 10^3\ kg\ m^{-3}$

Velocity of longitudinal waves in aluminium is
$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$
$$= \sqrt{\frac{7.5 \times 10^{10} + \frac{4}{3} \times 2.1 \times 10^{10}}{2.7 \times 10^3}}$$
$$= \mathbf{6.18 \times 10^3\ ms^{-1}}.$$

EXAMPLE 9. *For a steel rod, the Young's modulus of elasticity is 2.9×10^{11} Nm^{-2} and density is 8×10^3 $kg\ m^{-3}$. Find the velocity of the longitudinal waves in the steel rod.*

Solution. Here $Y = 2.9 \times 10^{11}\ Nm^{-2}$,

$\rho = 8 \times 10^3\ kg\ m^{-3}$

Velocity of longitudinal waves in steel is
$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.9 \times 10^{11}}{8 \times 10^3}} = \mathbf{6.02 \times 10^3\ ms^{-1}}.$$

EXAMPLE 10. *At a pressure of 10^5 Nm^{-2}, the volume strain of water is 5×10^{-5}. Calculate the speed of sound in water. Density of water is 10^3 $kg\ m^{-3}$.*

Solution. Bulk modulus of water is
$$\kappa = \frac{\text{Normal stress (pressure)}}{\text{Volume strain}}$$
$$= \frac{10^5}{5 \times 10^{-5}} = 2 \times 10^9\ Nm^{-2}$$

Density, $\rho = 10^3\ kg\ m^{-3}$

Speed of sound in water is
$$v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{2 \times 10^9}{10^3}} = \mathbf{1.414 \times 10^3\ ms^{-1}}.$$

EXAMPLE 11. *Estimate the speed of sound in air at standard temperature and pressure by using (i) Newton's formula and (ii) Laplace formula. The mass of 1 mole of air $= 29.0 \times 10^{-3}$ kg. For air, $\gamma = 1.4$.* **[NCERT]**

Solution. Density of air,
$$\rho = \frac{\text{Mass of 1 mole of air}}{\text{Volume of 1 mole of air}}$$
$$= \frac{29.0 \times 10^{-3}\ kg}{22.4\ \text{litre}}$$
$$= \frac{29.0 \times 10^{-3}\ kg}{22.4 \times 10^{-3}\ m^3} = 1.29\ kg\,m^{-3}$$

Standard pressure, $P = 1.01 \times 10^5$ Pa.

(*i*) According to Newton's formula, speed of sound in air at S.T.P. is
$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.01 \times 10^5}{1.29}} = \mathbf{280\ ms^{-1}}.$$

(*ii*) According to Laplace formula, speed of sound in air at S.T.P. is
$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}} = \mathbf{331.5\ ms^{-1}}.$$

✖ PROBLEMS FOR PRACTICE

1. The speed of sound in a liquid is 1500 ms^{-1}. The density of the liquid is 1.0×10^3 kg m^{-3}. Determine the bulk modulus of elasticity of the liquid.

(**Ans.** $2.25 \times 10^9\ Nm^{-2}$)

2. The longitudinal waves starting from a ship return from the bottom of the sea to the ship after 2.64 s. If the bulk modulus of water be 220 kg mm^{-2} and the density 1.1×10^3 kg m^{-3}, calculate the depth of the sea. Take $g = 9.8$ N kg^{-1}. **(Ans.** 1848 m)
3. At 10^5 Nm^{-2} atmospheric pressure the density of air is 1.29 kg m^{-3}. If $\gamma = 1.41$ for air, calculate the speed of sound in air. **(Ans.** 330.6 ms^{-1})
4. At normal temperature and pressure, 4 g of helium occupies a volume of 22.4 litre. Determine the speed of sound in helium. For helium, $\gamma = 1.67$ and 1 atmospheric pressure = 10^5 Nm^{-2}. **(Ans.** 967 ms^{-1})

✖ HINTS

2. Here $\kappa = 220 \text{ kg mm}^{-2} = 220 \times 10^6 \text{ kg m}^{-2}$

$$= 220 \times 9.8 \times 10^6 \text{ Nm}^{-2}$$

$$\rho = 1.1 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{220 \times 9.8 \times 10^6}{1.1 \times 10^3}} = 1400 \text{ ms}^{-1}$$

Depth of the sea $= \dfrac{vt}{2}$

$$= \frac{1400 \times 2.64}{2} = 1848 \text{ m.}$$

4. Density of helium,

$$\rho = \frac{4 \text{ g}}{22.4 \text{ litre}} = \frac{4 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3} = \frac{4}{22.4} \text{ kg m}^{-3}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.67 \times 10^5 \times 22.4}{4}} = 967 \text{ ms}^{-1}.$$

15.8 FACTORS AFFECTING SPEED OF SOUND IN A GAS

12. *Discuss the various factors which affect the speed of sound in a gas.*

Factors affecting the speed of sound in a gas. The factors such as density of a gas, its pressure, temperature, presence of moisture, etc., affect the speed of sound in a gaseous medium.

(*i*) **Effect of pressure.** The speed of sound in a gas is given by the Laplace formula,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature,

$$PV = \text{constant}$$

or

$$\frac{Pm}{\rho} = \text{constant} \qquad \left[\because \rho = \frac{m}{V} \text{ or } V = \frac{m}{\rho}\right]$$

Since m is a constant, so

$$\frac{P}{\rho} = \text{constant}$$

i.e., when pressure changes, density also changes in the same ratio so that the factor P/ρ remains unchanged. Hence pressure has no effect on the speed of sound in a gas.

(*ii*) **Effect of density.** Suppose two gases have the same pressure P and same value of γ (both are either monoatomic, diatomic or triatomic). If ρ_1 and ρ_2 are the densities of the two gases, then the speeds of sound in them will be

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Hence *at constant pressure, the speed of sound in a gas is inversely proportional to the square root of its density*. For example, the density of oxygen is 16 times the density of hydrogen.

$$\therefore \quad \frac{v_H}{v_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

or $\quad v_H = 4v_O$

i.e., the speed of sound in hydrogen is four times the speed of sound in oxygen.

(*iii*) **Effect of humidity.** The speed of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \textit{i.e.,} \quad v \propto \frac{1}{\sqrt{\rho}}$$

As the density of water vapour (0.8 kgm^{-3} at STP) is less than that of dry air (1.293 kgm^{-3} at STP), so the presence of moisture in air decreases the density of air. Since, the speed of sound is inversely proportional to the square root of density, so sound travels faster in moist air than in dry air.

(*iv*) **Effect of temperature.** For one mole of a gas, $PV = RT$. If M is the molecular mass of the gas and ρ its density, then

$$\rho = \frac{M}{V} \quad \text{or} \quad V = \frac{M}{\rho}$$

$$\therefore \quad \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M}$$

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

Clearly, $\quad v \propto \sqrt{T}$

Hence *the speed of sound in a gas is directly proportional to the square root of its absolute temperature.*

Temperature coefficient for the speed of sound in air. *It is defined as the increase in the velocity of sound for 1°C (or 1 K) rise in temperature of the gas.*

As $v \propto \sqrt{T}$ and $T(K) = t°C + 273$

$\therefore$ At 0°C, speed $v_0 \propto \sqrt{0+273}$

At t°C, speed $v_t \propto \sqrt{t+273}$

Hence $\frac{v_t}{v_0} = \sqrt{\frac{t+273}{0+273}}$

or $$v_t = v_0\left[1+\frac{t}{273}\right]^{1/2} = v_0\left[1+\frac{1}{2}\cdot\frac{t}{273}\right]$$

or $$v_t = v_0 + \frac{v_0 \times t}{546}$$

But speed of sound in air at 0°C,

$$v_0 = 332 \text{ ms}^{-1}$$

$$\therefore \quad v_t - v_0 = \frac{332 \times t}{546} = 0.61\, t$$

When $t = 1°C$, $v_t - v_0 = 0.61 \text{ ms}^{-1} = 61 \text{ cm s}^{-1}$

Hence the *velocity of sound in air increases* by 61 cm s^{-1} *for every* 1°C *rise of* temperature. This is known as temperature coefficient for sound in air.

(*v*) **Effect of wind.** As the sound is carried by air, so its velocity is affected by the wind velocity. Suppose the wind travels with velocity w at angle θ with direction of propagation of sound, as shown in Fig. 15.7. Clearly, the component of wind velocity in the direction of sound is $w \cos\theta$.

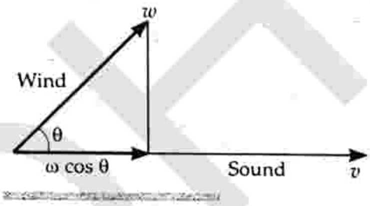


Fig. 15.7 Effect of wind.

$\therefore$ Resultant velocity of sound $= v + w\cos\theta$

When the wind blows in the direction of sound ($\theta = 0°$), resultant velocity $= v + w$.

When the wind blows in the opposite direction of sound ($\theta = 180°$), resultant velocity $= v - w$.

(*vi*) **Effect of frequency.** The speed of sound in air is independent of its frequency. Sound waves of different frequencies travel with the same speed in air, though their wavelengths in air are different. If the speed of sound were dependent on the frequency, we could not have enjoyed orchestra.

(*vii*) **Effect of amplitude.** To a large extent, the speed of sound is independent of the amplitude of the sound wave. But if the amplitude is very large, the compressions and rarefactions may cause large temperature variations which may affect the speed of sound.

Examples based on
Factors affecting Velocity of Sound through Gases

FORMULAE USED

1. *Effect of pressure.* There is no effect of pressure on velocity of sound.
2. *Effect of density* $v \propto \frac{1}{\sqrt{\rho}}$

   or $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
3. *Effect of temperature* $v \propto \sqrt{T}$

   or $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

   Also $v = \sqrt{\frac{\gamma RT}{M}}$,

   where M = molecular mass of the gas.
4. *Temperature coefficient of sound.* It is given by

   $$\alpha = \frac{v_t - v_0}{t}$$

   For air, $\alpha = 0.61 \text{ ms}^{-1}\,°C^{-1}$.

UNITS USED

Density ρ is in kg m^{-3}, pressure P in Nm^{-2}, temperature T in kelvin (K) and velocity v in ms^{-1}

EXAMPLE 12. *At what temperature will the speed of sound be double its value at 273 K ?* **[Himachal 09]**

Or

Calculate the temperature at which the speed of sound will be two times its value at 0°C. **[Delhi 02]**

Solution. $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$

Given $v_2 = 2v_1$, $T_1 = 273$ K

$$\therefore \quad \frac{2v_1}{v_1} = \sqrt{\frac{T_2}{273}} \quad \text{or} \quad \frac{T_2}{273} = 4$$

$$\therefore \quad T_2 = 4 \times 273 = \mathbf{1092\ K.}$$

EXAMPLE 13. *A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at S.T.P. Calculate the increase in wavelength, when temperature of air is 27°C.*

Solution. Here $\nu = 220$ Hz, $T_0 = 273$ K,

$$T = 273 + 27 = 300 \text{ K}, \quad \lambda_0 = 1.5 \text{ m}$$

Speed of sound at S.T.P. is

$$v_0 = \nu\lambda_0 = 220 \times 1.5 = 330 \text{ ms}^{-1}$$

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{300}{273}}$$

$$\therefore \quad v = \sqrt{\frac{300}{273}}\, v_0 = \sqrt{\frac{300}{273}} \times 330 \text{ ms}^{-1}$$

But $\quad v = \nu\lambda = 220 \times \lambda$

$$\therefore \quad 220 \times \lambda = \sqrt{\frac{300}{273}} \times 330$$

or $$\lambda = \sqrt{\frac{300}{273}} \times \frac{330}{220} = 1.57 \text{ m}$$

Increase in wavelength

$$= \lambda - \lambda_0 = 1.57 - 1.5 = \mathbf{0.07\ m.}$$

EXAMPLE 14. *Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000°C. Density of oxygen is 16 times that of hydrogen.*

Solution. Let v_0, v_{1000} = velocity of sound in oxygen at 0°C and 1000°C

v'_0, v'_t = velocities of sound in hydrogen at 0°C and t°C

For oxygen, $$\frac{v_{1000}}{v_0} = \sqrt{\frac{273+1000}{273}} = \sqrt{\frac{1273}{273}}$$

$$\therefore \quad v_{1000} = \sqrt{\frac{1273}{273}}\, v_0$$

Similarly, for hydrogen,

$$v'_t = \sqrt{\frac{273+t}{273}}\, v'_0$$

Given $v_{1000} = v'_t$

$$\therefore \quad \sqrt{\frac{1273}{273}}\, v_0 = \sqrt{\frac{273+t}{273}}\, v'_0.$$

or $$\sqrt{\frac{273+t}{1273}} = \frac{v_0}{v'_0} = \sqrt{\frac{\rho_H}{\rho_O}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\frac{273+t}{1273} = \frac{1}{16}$$

or $$273 + t = \frac{1273}{16} = 79.56$$

or $$t = 79.56 - 273 = \mathbf{-193.44°C}.$$

EXAMPLE 15. *Speed of sound in air is 332 ms^{-1} at S.T.P. What will be its value in hydrogen at S.T.P, if density of hydrogen at S.T.P. is $1/16^{th}$ that of air ?* **[MNREC 95]**

Solution. Speed sound, $v = \sqrt{\dfrac{\gamma P}{\rho}}$

Taking γ and P same for air and hydrogen, we can write

$$\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho_H}} = \sqrt{\frac{\rho_a}{(1/16)\rho_a}} = 4$$

$$\therefore \quad v_H = 4\, v_a = 4 \times 332 = \mathbf{1328\ ms^{-1}}.$$

EXAMPLE 16. *At normal temperature and pressure the speed of sound in air is 332 ms^{-1}. What will be the speed of sound in hydrogen at 546°C and 3 atmospheric pressure ? Air is 16 times heavier than hydrogen.*

Solution. First we find out speed of sound in hydrogen at normal temperature and pressure.

$$\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho_H}} = \sqrt{\frac{\rho_a}{(1/16)\rho_H}} = 4$$

$$\therefore \quad v_H = 4 \times 332 = 1328 \text{ ms}^{-1}$$

As the speed of sound is not affected by the change of pressure, we determine the effect of temperature alone. Let v_0 and v_{546} be the speeds of sound in hydrogen at 0°C and 546°C respectively. Then

$$\frac{v_{546}}{v_0} = \sqrt{\frac{273+546}{273+0}}$$

$$= \sqrt{\frac{819}{273}} = \sqrt{3} = 1.732$$

$$\therefore \quad v_{546} = 1.732\, v_0 = 1.732 \times 1328$$

$$= \mathbf{2300\ ms^{-1}}.$$

EXAMPLE 17. *Find the ratio of velocity of sound in hydrogen gas ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature. Given that molecular weights of hydrogen and helium are 2 and 4 respectively.* **[IIT 85]**

Solution. $v = \sqrt{\dfrac{\gamma RT}{M}}$

At constant temperature,

$$\frac{v_H}{v_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \cdot \frac{M_{He}}{M_H}}$$

$$= \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{2}} = \sqrt{\frac{42}{25}} = \mathbf{1.68.}$$

EXAMPLE 18. *The ratio of densities of oxygen and nitrogen is 16 : 14. At what temperature, the speed of sound in oxygen will be equal to its speed in nitrogen at 14°C ?*

Solution. Speed of sound,

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma R(t+273)}{M}}$$

Speed of sound in oxygen at t°C

$$= \sqrt{\frac{\gamma R(t+273)}{M_O}}$$

Speed of sound in nitrogen at 14°C

$$= \sqrt{\frac{\gamma R(14+273)}{M_N}}$$

But these two speeds have to be equal. Also, γ is same for both gases.

Hence

$$\sqrt{\frac{\gamma R(t+273)}{M_O}} = \sqrt{\frac{\gamma R(14+273)}{M_N}}$$

or $$\frac{M_O}{M_N} = \frac{t+273}{287}$$

Given $$\frac{M_O}{M_N} = \frac{16}{14}$$

$\therefore$ $$\frac{16}{14} = \frac{t+273}{287}$$

On solving, $t = \mathbf{55°C.}$

Example 19. *A gas is a mixture of two parts by volume of hydrogen and one part by volume of nitrogen. If the velocity of sound in hydrogen at 0°C is 1300 ms⁻¹, find the velocity of sound in the gaseous mixture at 27°C.*

Solution. Both hydrogen and nitrogen are diatomic gases. So the value of γ can be taken same for hydrogen, nitrogen and the mixture of gases.

$$\text{Density of mixture} = \frac{\text{Total mass}}{\text{Total volume}}$$

or $$\rho_{mix} = \frac{2V \times \rho_H + V \times \rho_N}{2V + V}$$

$$= \frac{2V \times \rho_H + V \times 14\rho_H}{3V} = \frac{16\rho_H}{3}$$

$[\because \rho_N = 14\rho_H]$

Velocity of sound in the mixture at 0°C,

$$v_0 = \sqrt{\frac{\gamma P}{\rho_{mix}}} = \sqrt{\frac{\gamma P \times 3}{16\rho_H}}$$

Velocity of sound in hydrogen at 0°C

$$v_H = \sqrt{\frac{\gamma P}{\rho_H}}$$

$\therefore$ $$\frac{v_0}{v_H} = \sqrt{\frac{\gamma P \times 3}{16\rho_H}} \times \sqrt{\frac{\rho_H}{\gamma P}} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$$

or $$v_0 = \frac{\sqrt{3}}{4} v_H = \frac{\sqrt{3}}{4} \times 1300 = 325\sqrt{3} \text{ ms}^{-1}$$

Velocity of sound in the mixture at 27°C,

$$v_t = v_0\left(1 + \frac{t}{546}\right)$$

$$= 325\sqrt{3}\left(1 + \frac{27}{546}\right) = \mathbf{591 \text{ ms}^{-1}}.$$

Problems For Practice

1. Find the temperature at which the velocity of sound in air will be $1\frac{1}{2}$ times the velocity at 11°C. **(Ans.** 366°C)
2. The speed of sound in air is 332 ms^{-1} at 0°C. At what temperature will the speed become one half of that at 0°C ? **[Himachal 08]** **(Ans.** –204.75°C)
3. What is the ratio of the velocity of sound in hydrogen ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature ? **[IIT 85]** **(Ans.** $\sqrt{42}/5$)
4. An observer sets his watch by the sound of a signal fired from a tower yet he finds that his watch is slow by 5 s. Find the distance of the tower from the observer. The temperature of air during the observation is 20°C and the velocity of sound in air at 0°C is 332 ms^{-1}. **(Ans.** 1720 m)
5. A sound wave propagating in air has a frequency of 4000 Hz. Calculate the percentage change in wavelength when the wavefront, initially in a region where T = 27°C, enters a region where the temperature decreases to 10°C. **(Ans.** 3%)
6. At what temperature will the velocity of sound in hydrogen be the same as in oxygen at 100°C ? Density of oxygen is 16 times the density of hydrogen. **(Ans.** – 249.7°C)
7. The speed of sound in dry air at S.T.P. is 332 ms^{-1}. Assuming air as composed of 4 parts of nitrogen and one part of oxygen, calculate velocity of sound in oxygen under similar conditions, when the densities of oxygen and nitrogen at S.T.P. are in the ratio of 16 : 14. **(Ans.** 314.77 ms^{-1})

Hints

1. Given $v_t = \frac{3}{2} v_{11}$

or $$v_0\sqrt{\frac{273+t}{273}} = \frac{3}{2} v_0 \sqrt{\frac{273+11}{273}}$$

On squaring, $$\frac{273+t}{273} = \frac{9}{4} \times \frac{284}{273}$$

On solving, $t = \mathbf{366°C.}$

3. Speed of sound, $v = \sqrt{\frac{\gamma RT}{M}}$

$\therefore$ $$\frac{v_H}{v_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \cdot \frac{M_{He}}{M_H}} = \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{2}} = \frac{\sqrt{42}}{5}.$$

4. $$v_{20} = v_0\sqrt{\frac{273+20}{273}} = 332\sqrt{\frac{293}{273}} = 340 \text{ ms}^{-1}$$

Distance of tower from the observer

$= 340 \times 5 = \mathbf{1720 \text{ m.}}$

5. $$\frac{v_2}{v_1} = \sqrt{\frac{273+10}{273+27}} = \sqrt{\frac{283}{300}} = 0.97$$

As frequency remains unchanged, so

$$\frac{\lambda_2}{\lambda_1} = \frac{\nu\lambda_2}{\nu\lambda_1} = \frac{v_2}{v_1} = 0.97$$

Percentage change in wavelength,

$$\frac{\lambda_1 - \lambda_2}{\lambda_1} \times 100 = \left(1 - \frac{\lambda_2}{\lambda_1}\right) \times 100$$

$$= (1 - 0.97) \times 100 = \mathbf{3\%}.$$

6. Given : $(v_H)_t = (v_O)_{100}$

$$(v_H)_0 \cdot \sqrt{\frac{273 + t}{273}} = (v_O)_0 \sqrt{\frac{273 + 100}{273}}$$

$$\left(\frac{v_H}{v_O}\right)_0 = \sqrt{\frac{273 + 100}{273 + t}} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16}{1}} = \frac{4}{1}$$

or $$\frac{373}{273 + t} = 16$$

On solving, $t = \mathbf{-249.7°C}$.

7. Density of mixture $= \dfrac{\text{Total mass}}{\text{Total volume}}$

$$\rho_{mix} = \frac{4V \times \rho_N + V \times \rho_O}{4V + V}$$

$$= \frac{\rho_O\left(4\frac{\rho_N}{\rho_O} + 1\right)}{5} = \frac{\rho_O\left(4 \times \frac{14}{16} + 1\right)}{5}$$

$$= \frac{9}{10}\rho_O = 0.9\,\rho_O$$

$$\frac{v_O}{v_{mix}} = \sqrt{\frac{\rho_{mix}}{\rho_O}} = \sqrt{\frac{0.9\,\rho_O}{\rho_O}} = \sqrt{0.9} = 0.9487$$

$$\therefore\ v_O = 0.9487 \times v_{mix} = 0.9487 \times 332 = \mathbf{314.77\ ms^{-1}}.$$

15.9 DISPLACEMENT RELATION FOR A PROGRESSIVE WAVE

13. *What is a progressive wave ? What is a plane progressive harmonic wave ? Establish the displacement relation for harmonic wave travelling along the positive direction of X-axis.*

Progressive wave. *A wave that travels from one point of the medium to another is called a progressive wave.* A progressive wave may be transverse or longitudinal.

Plane progressive harmonic wave. *If during the propagation of a wave through a medium, the particles of the medium vibrate simple harmonically about their mean positions, then the wave is said to be plane progressive harmonic wave.* In a harmonic progressive wave of given frequency, all particles have same amplitude but the phase of oscillation changes from one particle to the next.

Displacement relation for a progressive harmonic wave. Suppose a simple harmonic wave starts from the origin O and travels along the positive direction of X-axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at the origin O $(x = 0)$ at any instant t is given by

$$y(0, t) = A \sin \omega t \qquad ...(1)$$

where T is the periodic time and A the amplitude of the wave.

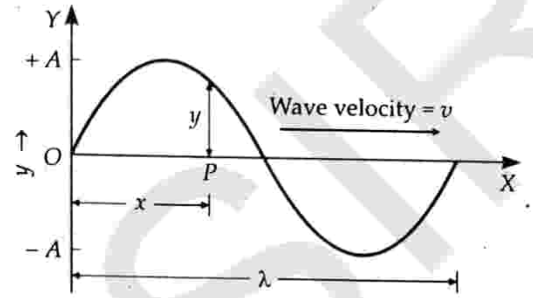


Fig. 15.8 A simple harmonic wave.

Consider a particle P on the X-axis at a distance x from O. The disturbance starting from the origin O will reach P in x/v seconds. This means the particle P will start vibrating x/v seconds later than the particle at O. Therefore,

Displacement of the particle at P at any instant t
= Displacement of the particle at O at a time x/v seconds earlier
= Displacement of the particle at O at time $(t - x/v)$.

Thus the displacement of the particle at P at any time t can be obtained by replacing t by $(t - x/v)$ in the equation (1). It is given by

$$y(x, t) = A \sin \omega\left(t - \frac{x}{v}\right) = A \sin\left(\omega t - \frac{\omega}{v}x\right)$$

But $$\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$$

The quantity $k = 2\pi/\lambda$ is called *angular wave number* or *propagation constant*. Hence

$$y(x, t) = A \sin(\omega t - kx) \qquad ...(2)$$

This equation represents a harmonic wave travelling along the positive direction of the X-axis. It can also be written in the following forms :

$$y(x, t) = A \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

or $$y(x, t) = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad ...(3)$$

$$= A \sin\frac{2\pi}{T}\left(t - \frac{x}{\lambda}T\right)$$

But $$\frac{\lambda}{T} = v$$

$$\therefore \quad y(x, t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \quad ...(4)$$

Also $y(x, t) = A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right)$

or $$y(x, t) = A \sin \frac{2\pi}{\lambda}(vt - x) \quad ...(5)$$

Equations (2), (3), (4) and (5) are the various forms of plane progressive wave. If the initial phase of the particle at O is ϕ_0, then the equation of wave motion will be

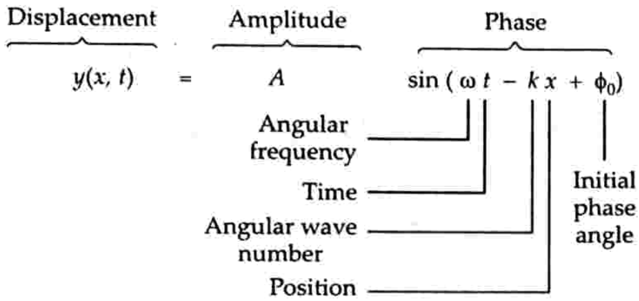


Fig. 15.9

A harmonic wave travelling along negative direction of X-axis can be written as

$$y(x, t) = A \sin(\omega t + kx + \phi_0)$$

15.10 PHASE AND PHASE DIFFERENCE

14. *What do you mean by phase of a wave ? Discuss the phase change with time and position.*

Phase of a wave. *The phase of a harmonic wave is a quantity that gives complete information of the wave at any time and at any position.* It is equal to the argument of the sine or cosine function representing the wave. Suppose a harmonic wave is given by

$$y(x, t) = A \sin(\omega t - kx + \phi_0) \quad ...(1)$$

Then the phase of the wave at position x and time t is given by

$$\phi = \omega t - kx + \phi_0 \quad ...(2)$$

Clearly, the phase of a wave is periodic both in time and space. At a given point (x = constant), the phase changes with time t and at a given instant (t = constant), it changes with distance x.

Phase change with time. Taking x as constant, if we differentiate equation (2) w.r.t., time t, we get

$$\frac{\Delta\phi}{\Delta t} = \omega$$

Thus the phase change at a given position (x = constant) in time Δt is given by

$$\Delta\phi = \omega\,\Delta t = \frac{2\pi}{T}\,\Delta t$$

Hence *we can define the time period of a wave as the time in which the phase of a particle of the medium changes by* 2π.

Phase change with position. Taking t as constant, if we differentiate equation (2) w.r.t. position x, we get

$$\frac{\Delta\phi}{\Delta x} = -k$$

Thus the phase difference, at any instant of time t, between two particles separated by distance Δx is given by

$$\Delta\phi = -k\,\Delta x = -\frac{2\pi}{\lambda}\,\Delta x$$

Hence *we can define the wavelength of a wave as the distance between two points (or particles) which have a phase difference of* 2π *at any given instant.* The negative sign indicates that farther the particle is located from the origin in the positive X-direction, the more it lags behind in phase.

15.11 PARTICLE VELOCITY AND ACCELERATION

15. *For a simple harmonic wave, deduce expressions for (a) particle velocity and (b) particle acceleration. Discuss their phase relationship with displacement.*

(*a*) **Particle velocity.** The particle velocity V is different from the wave velocity v. It *is the velocity with which the particles of the medium vibrate about their mean positions.*

The displacement relation for a harmonic wave travelling along positive X-direction is

$$y(x, t) = A \sin(\omega t - kx) \quad ...(1)$$

Differentiating (1) w.r.t. time t, and taking x constant, we get the particle velocity

$$V = \frac{dy}{dt} = \omega A \cos(\omega t - kx) \quad ...(2)$$

or $$V = \omega A \sin[(\omega t - kx) + \pi/2] \quad ...(3)$$

It may be noted that

(*i*) While the wave velocity ($v = \nu\lambda$) remains constant, the particle velocity changes simple harmonically with time.

(*ii*) The particle velocity is ahead of displacement in phase by $\pi/2$ radian.

(*iii*) The maximum particle velocity or the *velocity amplitude* is

$$V_0 = \omega A = \frac{2\pi}{T} A$$

$$= \frac{2\pi}{T} \textit{ times the displacement amplitude } A.$$

(*iv*) If we differentiate equation (1) w.r.t. position x, we get

$$\frac{dy}{dx} = -kA \cos(\omega t - kx) \quad ...(4)$$

From equations (2) and (4), we get

$$\frac{V}{dy/dx} = \frac{\omega A \cos(\omega t - kx)}{-kA\cos(\omega t - kx)}$$

$$= -\frac{\omega}{k} = -\frac{2\pi\nu}{2\pi/\lambda} = -v \quad \text{or} \quad V = -v\frac{dy}{dx}.$$

∴ Particle velocity at a point = – Wave velocity × slope of displacement curve at that point.

(*b*) **Particle acceleration.** If we differentiate equation (2) with respect to time *t*, we get the particle acceleration

$$a = \frac{dV}{dt} = -\omega^2 A \sin(\omega t - kx) = -\omega^2 y$$

or
$$a = \omega^2 A \sin[(\omega t - kx) + \pi]$$

It may be noted that

(*i*) The maximum value of particle acceleration or the *acceleration amplitude* is

$$a_0 = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$$

$$= \left(\frac{2\pi}{T}\right)^2 \textit{ times the displacement amplitude.}$$

(*ii*) The particle acceleration is ahead of the particle displacement in phase by π radian.

15.12 SPEED OF A TRAVELLING WAVE

16. *Define wave velocity or phase velocity. Deduce its relation with angular frequency* ω *and propagation constant* k.

Wave velocity or phase velocity. *The distance covered by a wave in the direction of its propagation per unit time is called the wave velocity.* It represents the velocity with which a disturbance is transferred from one particle to the next with the actual motion of the particles.

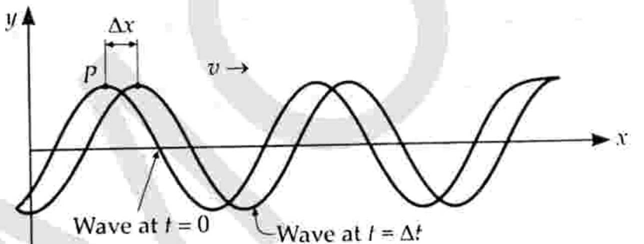


Fig. 15.10 Plot of a harmonic wave at $t = 0$ and $t = \Delta t$.

Fig. 15.10 shows two plots of the harmonic wave $y = A\sin(\omega t - kx)$ at two different instants of time t and $t + \Delta t$. During the small time interval Δt, the entire wave pattern moves through distance Δx in the positive *X*-direction. As the wave moves, each point of the moving waveform, such as point *P* marked on the peak retains its displacement *y*. This is possible only when the phase of the wave remains constant.

∴ $\omega t - kx$ = constant.

Differentiating both sides w.r.t., time *t*, we get

$$\omega - k\frac{dx}{dt} = 0 \quad \text{or} \quad \frac{dx}{dt} = \frac{\omega}{k}$$

But $\frac{dx}{dt}$ = wave velocity, v

$$\therefore \quad v = \frac{\omega}{k} = \frac{\lambda}{T} = \nu\lambda \qquad \left[\because \ \omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}\right]$$

Examples based on Progressive Waves

FORMULAE USED

1. A plane progressive harmonic wave travelling along positive direction of *X*-axis can be represented by any of the following expressions :

   (*i*) $y = A\sin(\omega t - kx), \quad k = 2\pi/\lambda$

   (*ii*) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

   (*iii*) $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

   where λ is the wavelength, v is the velocity, A the amplitude and x is the distance of observation point from the origin.

2. For a progressive wave travelling along –ve X-axis,

   $$y = A\sin(\omega t + kx)$$

   or $$y = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) = A\sin\frac{2\pi}{\lambda}(vt + x)$$

3. Phase, $\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi_0$,

   where ϕ_0 is the initial phase.

4. Phase change with time, $\Delta\phi = \frac{2\pi}{T}\Delta t$

5. Phase change with position, $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

6. Instantaneous particle velocity

   $$u = \frac{dy}{dt} = \frac{2\pi A}{T}\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

   Velocity amplitude, $u_0 = \frac{2\pi A}{T} = \omega A$

7. Instantaneous particle acceleration

   $$f = \frac{du}{dt} = -\frac{4\pi^2}{T^2} A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = -\omega^2 y$$

   Acceleration amplitude, $f_0 = \frac{4\pi^2}{T^2} A = \omega^2 A$

UNITS USED

Displacement y and amplitude A have same units m or cm. If λ and x are in m, wave velocity v is in ms^{-1}. Propagation constant $k = 2\pi/\lambda$ is in rad m^{-1}.

EXAMPLE 20. *The displacement y of a particle in a medium can be expressed as*

$$y = 10^{-6} \sin(100\,t + 20x + \pi/4)$$

where t is in second and x in metre. What is the speed of the wave ? **[AIEEE 04]**

Solution. We compare the given wave equation with the standard wave equation,

$$y = A \sin(\omega t + kx + \phi)$$

We get $\omega = 100 \text{ rad s}^{-1}$ and $k = 20 \text{ rad m}^{-1}$.

Speed of the wave,

$$v = \frac{\omega}{k} = \frac{100}{20} = \mathbf{20\ ms^{-1}}.$$

EXAMPLE 21. *A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3 ms^{-1} and 90 ms^{-2} respectively. Velocity of the wave is 20 ms^{-1}. Find the waveform.* **[IIT 05]**

Solution. Here $v_{max} = \omega A = 3 \text{ ms}^{-1}$

$$a_{max} = \omega^2 A = 90 \text{ ms}^{-2}$$

Velocity of the wave,

$$v = \frac{\omega}{k} = 20 \text{ ms}^{-1}$$

Clearly, $A = \frac{\omega^2 A^2}{\omega^2 A} = \frac{(3)^2}{90} = 0.1 \text{ m}$

$$\omega = \frac{\omega^2 A}{\omega A} = \frac{90}{3} = 30 \text{ rad s}^{-1}$$

$$k = \frac{\omega}{\omega/k} = \frac{30}{20} = 1.5 \text{ rad m}^{-1}$$

The equation for the waveform is

$$y = A \sin(\omega t + kx) = \mathbf{0.1 \sin(30t + 1.5x)}.$$

EXAMPLE 22. *A wave travelling along a string is described by*

$$y(x, t) = 0.005 \sin(80.0\,x - 3.0\,t),$$

in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s. **[NCERT ; Central Schools 05]**

Solution. Given $y(x, t) = 0.005 \sin(80.0\,x - 3.0\,t)$

The displacement equation for a harmonic wave is

$$y(x, t) = A \sin\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right)$$

On comparing the above two equations, we get

$$A = 0.005 \text{ m}, \frac{2\pi}{\lambda} = 80.0 \text{ rad m}^{-1}, \frac{2\pi}{T} = 3.0 \text{ rad s}^{-1}.$$

(*a*) Amplitude, $A = \mathbf{0.005\ m}$.

(*b*) Wavelength,

$$\lambda = \frac{2\pi \text{ rad}}{80.0 \text{ rad m}^{-1}} = 7.85 \times 10^{-2} \text{ m} = \mathbf{7.85\ cm}.$$

(*c*) Time period,

$$T = \frac{2\pi \text{ rad}}{3.0 \text{ rad s}^{-1}} = \mathbf{2.09\ s}.$$

Frequency,

$$\nu = \frac{1}{T} = \frac{1}{2.09 \text{ s}} = \mathbf{0.48\ Hz}.$$

Displacement of the wave at a distance $x = 30.0$ cm and time $t = 20$ s,

$$y = (0.005 \text{ m}) \sin(80.0 \times 0.3 - 3.0 \times 20)$$
$$= (0.005 \text{ m}) \sin(-36 \text{ rad}) = \textbf{zero}.$$

EXAMPLE 23. *The equation of a plane progressive wave is*

$$y = 10 \sin 2\pi(t - 0.005\,x)$$

where y and x are in cm and t in seconds. Calculate the amplitude, frequency, wavelength and velocity of the wave. **[Delhi 99]**

Solution. Given : $y = 10 \sin 2\pi(t - 0.005\,x)$ ...(1)

The standard equation for a harmonic wave is

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(2)$$

Comparing equations (1) and (2), we get

$$A = 10, \ \frac{1}{T} = 1, \ \frac{1}{\lambda} = 0.005$$

(*i*) Amplitude, $A = \mathbf{10\ cm}$.

[y and A have same units]

(*ii*) Frequency, $\nu = \frac{1}{T} = \mathbf{1\ Hz}$.

(*iii*) Wavelength, $\lambda = \frac{1}{0.005} = \mathbf{200\ cm}$.

[x and λ have same units]

(*iv*) Velocity, $v = \nu\lambda = 1 \times 200 = \mathbf{200\ cm\ s^{-1}}$.

Example 24. *A wave travelling along a string is described by equation $y(x,t) = 0.05\sin(40x - 5t)$ in which the numerical constants are in SI units (0.05 m, 40 rad m^{-1} and 5 rad s^{-1}). Calculate the displacement at distance 35 cm and time 10 sec.* **[Delhi 08]**

Solution. Given :

$$y(x,t) = 0.05\sin(40x - 5t)$$

We compare with the standard equation,

$$y(x,t) = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$$

$\therefore \quad A = 0.05$ m, $\frac{2\pi}{\lambda} = 40$ rad m^{-1}

$\frac{2\pi}{T} = 5$ rad s^{-1}.

(*a*) Amplitude, $A = \mathbf{0.05}$ **m.**

(*b*) Wavelength, $\lambda = \frac{2\pi \text{rad}}{40 \text{rad m}^{-1}} = \frac{\pi}{20}$ m $= \mathbf{15.7}$ **cm.**

(*c*) Time period, $T = \frac{2\pi \text{rad}}{5 \text{rad s}^{-1}} = \frac{2\pi}{5}$ s $= \mathbf{1.26}$ **cm.**

(*d*) Frequency, $\nu = \frac{1}{T} = \frac{5}{2\pi} = \mathbf{0.8}$ **Hz.**

At $x = 35$ cm $= 0.35$ m and $t = 10$ s, the displacement is

$$y = 0.05 \sin(40 \times 0.35 - 5 \times 10)$$
$$= 0.05 \sin(-36) \text{ m} = -0.05 \sin 36 \text{ m}.$$

Example 25. *A wave travelling along a string is given by*

$$y(x, t) = 0.005 \sin(80x - 3t)$$

where the numerical values are in SI units. Symbols have their usual meanings. Calculate :

(*a*) *Frequency of the wave.* (*b*) *Velocity of the wave.*

(*c*) *Amplitude of particle velocity.*

[Central Schools 08, 09]

Solution. Given : $y(x,t) = 0.005 \sin(80x - 3t)$

Also, $y(x,t) = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$

$\therefore \quad A = 0.005$ m,

$\frac{2\pi}{\lambda} = 80$ rad m^{-1},

$\frac{2\pi}{T} = 3$ rad s^{-1}.

(*a*) Frequency, $\nu = \frac{1}{T} = \frac{3}{2\pi} = \mathbf{0.48}$ **Hz.**

(*b*) $\lambda = \frac{2\pi}{80}$ m

Wave velocity,

$$v = \nu\lambda = \frac{3}{2\pi} \times \frac{2\pi}{80} = \frac{3}{80} \text{ ms}^{-1} = \mathbf{7.5 \ cms^{-1}}$$

(*c*) Amplitude of particle velocity

$$= \frac{2\pi}{T} A = 3 \times 0.005 = \mathbf{0.015 \ ms^{-1}}.$$

Example 26. *A displacement wave is represented by*

$$y = 0.25 \times 10^{-3} \sin(500\,t - 0.025\,x)$$

where y, t and x are in cm, sec and metres respectively. Deduce (i) amplitude (ii) period (iii) angular frequency, and (iv) wavelength. Also deduce the amplitude of particle velocity and particle acceleration. **[Delhi 03C]**

Solution. Given

$$y = 0.25 \times 10^{-3} \sin(500t - 0.025x)$$

Comparing it with standard equation :

$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$, we get

(*i*) Amplitude,

$A = \mathbf{0.25 \times 10^{-3}}$ **cm.**

(*ii*) $\frac{2\pi}{T} = 500$ or $T = \frac{2\pi}{500} = \frac{\pi}{250} = \mathbf{0.01257}$ **s.**

(*iii*) Angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} \times 250 = \mathbf{500 \ rad \ s^{-1}}.$$

(*iv*) $\frac{2\pi}{\lambda} = 0.025$ or $\lambda = \frac{2\pi}{0.025} = \mathbf{251.2}$ **m.**

(*v*) Velocity amplitude

$$= \omega A = 500 \times 0.25 \times 10^{-3} = \mathbf{0.125 \ cm \ s^{-1}}.$$

(*vi*) Acceleration amplitude

$$= \omega^2 A = (500)^2 \times 0.25 \times 10^{-3} = \mathbf{62.5 \ cm \ s^{-2}}.$$

Example 27. *The speed of a wave in a stretched string is 20 ms^{-1} and its frequency is 50 Hz. Calculate the phase difference in radian between two points situated at a distance of 10 cm on the string.*

Solution. Here $v = 20$ ms^{-1}, $\nu = 50$ Hz, $\Delta x = 10$ cm

Wavelength, $\lambda = \frac{v}{\nu} = \frac{20}{50} = 0.4$ m $= 40$ cm

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{40} \times 10 = \frac{\pi}{2}$ **rad.**

Example 28. *Write the equation of a progressive wave propagating along the positive x-direction, whose amplitude is 5 cm, frequency 250 Hz and velocity 500 ms^{-1}.*

Solution. Here $A = 5$ cm $= 0.05$ m, $\nu = 250$ Hz, $v = 500$ ms^{-1}

Wavelength, $\lambda = \frac{v}{\nu} = \frac{500}{250} = 2$ m,

Period, $T = \frac{1}{\nu} = \frac{1}{250}$ s

The equation for the given wave can be written as

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 0.05 \sin 2\pi\left(250t - \frac{x}{2}\right)$$

or $\quad y = \mathbf{0.05 \sin \pi (500\,t - x)}$ **metre.**

Example 29. *For the plane wave*

$$y = 2.5 \times 10^{-0.02x} \cos(800t - 0.82x + \pi/2),$$

write down

(*i*) *the general expression for phase* ϕ

(*ii*) *the phase at* $x = 0$, $t = 0$

(iii) the phase difference between the points separated by 20 cm along x-axis.

(iv) the change in phase at a given place 0.6 milli second and

(v) the amplitude at $x = 100$ m.

Take units of y, t, x as 10^{-5} cm, s and m respectively.

Solution. (*i*) Phase, $\phi = \mathbf{800t - 0.82x + \frac{\pi}{2}}$.

(*ii*) At $x = 0$, $t = 0$, $\phi = \pi/2$ **rad.**

(*iii*) Here $\Delta x = 20$ cm $= 0.20$ m. Therefore

$$\Delta\phi = -0.82\ \Delta x = -0.82 \times 0.20 = -\mathbf{0.164\ rad.}$$

(*iv*) Here $\Delta t = 0.6$ ms $= 0.6 \times 10^{-3}$ s. Therefore,

$$\Delta\phi = 800\ \Delta t = 800 \times 0.6 \times 10^{-3} = \mathbf{0.48\ rad.}$$

(*v*) At $x = 100$ m, the amplitude is

$$A = 2.5 \times 10^{-0.02x}$$

$$= 2.5 \times 10^{-0.02 \times 100} \times 10^{-5} \text{ cm}$$

$$= \mathbf{0.025 \times 10^{-5}\ cm.}$$

[$\because$ unit of A is 10^{-5} cm as that of y]

EXAMPLE 30. *A simple harmonic wave train of amplitude 1 cm and frequency 100 vibrations is travelling in positive x-direction with velocity 15 ms^{-1}. Calculate the displacement y, the particle velocity and particle acceleration at $x = 180$ cm from the origin at $t = 5$ s.*

Solution. Let the displacement of the wave be given by

$$y = A \sin \frac{2\pi}{\lambda}(vt - x) = A \sin \frac{2\pi\nu}{v}(vt - x)$$

$$\left[\because \lambda = \frac{v}{\nu}\right]$$

But $A = 1$ cm, $\nu = 100$ Hz,

$v = 15$ ms^{-1} $= 1500$ cm s^{-1},

$x = 180$ cm, $t = 5$ s

$$\therefore \quad y = 1 \sin \frac{2\pi \times 100}{1500}(1500 \times 5 - 180)$$

$$= \sin \frac{2\pi}{15} \times 7320 = \sin 2\pi \times 4800 = \mathbf{0.}$$

Particle velocity,

$$u = \frac{dy}{dt} = 2\pi A\nu \cos \frac{2\pi}{\lambda}(vt - x)$$

As calculated above,

$$\sin \frac{2\pi}{\lambda}(vt - x) = 0 \quad \text{at} \quad x = 180 \text{ cm and } t = 5 \text{ s}$$

$$\therefore \quad \cos \frac{2\pi}{\lambda}(vt - x) = 1$$

$\therefore$ Particle velocity,

$$u = 2\pi A\nu = 2\pi \times 1 \times 100 = \mathbf{200\pi\ cm\ s^{-1}.}$$

Particle acceleration,

$$f = -\frac{4\pi^2 v^2}{\lambda^2} y = \mathbf{0.} \qquad [\because y = 0]$$

EXAMPLE 31. *A certain spring has a linear mass density of 0.25 kg m^{-1} and is stretched with a tension of 25 N. One end is given a sinusoidal motion with frequency 5 Hz and amplitude 0.01 m. At time $t = 0$, the other end has zero displacement and is moving in the positive y-direction.*

(i) Find the wave speed, amplitude, angular frequency, period, wavelength and wave number.

(ii) Write a wave function representing the wave.

(iii) Find the position of the point at $x = 0.25$ m at time $t = 0.1$ s.

Solution. Here $m = 0.25$ kg m^{-1},

Tension $T = 25$ N, $\nu = 5$ Hz, $A = 0.01$ m

(*i*) Wave speed, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{25}{0.25}} = \mathbf{10\ ms^{-1}}$

Amplitude, $A = 0.01$ m

Angular frequency,

$$\omega = 2\pi\nu = 2 \times 3.14 \times 5 = \mathbf{31.4\ rad\ s^{-1}.}$$

Time period, $T = \frac{1}{\nu} = \frac{1}{5} = \mathbf{0.2\ s.}$

Wavelength, $\lambda = \frac{v}{\nu} = \frac{10}{5} = \mathbf{2\ m.}$

Wave number, $k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{2} = \mathbf{3.14\ m^{-1}.}$

(*ii*) The given wave can be represented by the wave function,

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$= 0.01 \sin 2\pi\left(\frac{t}{0.2} - \frac{x}{2}\right)$$

$$= 0.01 \sin (10\pi t - \pi x)$$

$$= \mathbf{0.01 \sin (31.4t - 3.14x).}$$

(*iii*) At $x = 0.25$ m and $t = 0.1$ s, the displacement is

$$y = 0.01 \sin 2\pi\left(\frac{0.1}{0.2} - \frac{0.25}{2}\right)$$

$$= 0.01 \sin 2\pi(0.5 - 0.125)$$

$$= 0.01 \sin (0.75\pi) = 0.01 \sin 135°$$

$$= 0.01 \times 0.707 = \mathbf{0.00707\ m.}$$

PROBLEMS FOR PRACTICE

1. A wave on a string is described by $y(x, t) = 0.005 \sin(6.28x - 314t)$, in which all quantities are in SI units. Calculate its (*i*) amplitude and (*ii*) wavelength. **[Central Schools 03]**

[Ans. (*i*) 0.005 m (*ii*) 1 m**]**

2. The equation of a transverse wave travelling along a coil spring is $y = 4.0 \sin \pi (0.010\,x - 2.0\,t)$ where y and x are in cm and t in s. Find the (*i*) amplitude (*ii*) wavelength (*iii*) initial phase at the origin (*iv*) speed and (*v*) frequency on the wave. [**Ans.** (*i*) 4.0×10^{-2} m (*ii*) 2.0 m (*iii*) 0 (*iv*) $2.0\ ms^{-1}$ (*v*) $1.0\,s^{-1}$]

3. The equation of transverse wave travelling in a rope is given by $y = 10 \sin \pi (0.01\,x - 2.00\,t)$ where y and x are in cm and t in seconds. Find the amplitude, frequency, velocity and wavelength of the wave. **[Delhi 97]**
(**Ans.** 10 cm, 1 Hz, 200 cm s^{-1}, 200 cm)

4. A simple harmonic wave is expressed by equation,
$$y = 7 \times 10^{-6} \sin \left(800\,\pi t - \frac{\pi}{42.5}\,x\right)$$
where y and x are in cm and t in seconds. Calculate the following : (*i*) amplitude (*ii*) frequency (*iii*) wave length (*iv*) wave velocity, and (*v*) phase difference between two particles separated by 17.0 cm. **[Delhi 05]**

5. For a travelling harmonic wave,
$$y = 2.0 \cos (10t - 0.0080\,x + 0.18)$$
where x and y are in cm and t is in seconds. What is the phase difference between two points separated by (*i*) a distance of 0.5 m and (*ii*) a time gap of 0.5 s ? [**Ans.** (*i*) – 0.4 rad (*ii*) 5 rad]

6. Find the displacement of an air particle 3.5 m from the origin of disturbance at $t = 0.05$ s, when a wave of amplitude 0.2 mm and frequency 500 Hz travels along it with a velocity 350 ms^{-1}. (**Ans.** 0)

7. A simple harmonic wave-train is travelling in a gas in the positive direction of the X-axis. Its amplitude is 2 cm, velocity 45 ms^{-1} and frequency 75 s^{-1}. Write down the equation of the wave. Find the displacement of the particle of the medium at a distance of 135 cm from the origin in the direction of the wave at the instant $t = 3$ s.
[**Ans.** $y = 2 \sin 2\pi \left(75\,t - \frac{x}{60}\right)$, – 2 cm]

8. The phase difference between the vibrations of two medium particles due to the transmission of a wave is $2\pi/3$. The distance between the particles is 15 cm. Determine the wavelength of the wave. (**Ans.** 45 cm)

9. The distance between two points on a stretched string is 20 cm. The frequency of the progressive wave is 400 Hz and velocity 100 ms^{-1}. Find the phase difference between these two points. (**Ans.** 1.6 π or 288°)

10. A sound-source of frequency 500 Hz is producing longitudinal waves in a spring. The distance between two consecutive rarefractions is 24 cm. If the amplitude of vibration of a particle of the spring is 3.0 cm and the wave is travelling in the negative x-direction, then write the equation for the wave. Assume that the source is at $x = 0$ and at this point the displacement is zero at the time $t = 0$.
[**Ans.** $y = 3.0 \sin 2\pi (25\,t + x/24)$]

Hints

1. Comparing $y = 0.005 \sin (6.28\,x - 314\,t)$
and $y = A \sin \left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right)$, we get $A = \mathbf{0.005\ m.}$
$$\frac{2\pi}{\lambda} = 6.28 \quad \text{or} \quad \lambda = \frac{2\pi}{6.28} = \frac{2 \times 3.14}{6.28} = \mathbf{1\ m.}$$

4. We compare the given equation with
$$y = A \sin \left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$
(*i*) $A = \mathbf{7 \times 10^{-6}\ cm.}$
(*ii*) $\frac{2\pi}{T} = 800\,\pi \quad \therefore \quad \nu = \frac{1}{T} = \mathbf{400\ Hz.}$
(*iii*) $\frac{2\pi}{\lambda} = \frac{\pi}{42.5} \quad \therefore \quad \lambda = 2 \times 42.5 = \mathbf{85\ cm.}$
(*iv*) $v = \nu\lambda = 400 \times 85 = 34000\ cm\ s^{-1} = \mathbf{340\ ms^{-1}}.$
(*v*) $\Delta\phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{85\ cm} \times 17.0\ cm = \mathbf{\frac{2\pi}{5}\ rad.}$

5. Given : $y = 2.0 \cos (10t - 0.0080\,x + 0.18)$
Comparing with $y = A \cos \left(\frac{2\pi t}{T} - \frac{2\pi}{\lambda} x + \phi_0\right)$,
we get,
$$A = 2.0\ cm, \quad \frac{2\pi}{T} = 10\,s^{-1}, \quad \frac{2\pi}{\lambda} = 0.0080\ cm^{-1}$$
(*i*) Here $\Delta x = 0.5$ m = 50 cm. The phase difference is
$$\Delta\phi = -\frac{2\pi}{\lambda} \Delta x = -0.0080 \times 50 = -\mathbf{0.4\ rad.}$$
(*ii*) Here $\Delta t = 0.5$ s. The phase difference is
$$\Delta\phi = \frac{2\pi}{T} \Delta t = 10 \times 0.5 = \mathbf{5\ rad.}$$

6. Let $y = A \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right]$
But $A = 0.2$ mm, $T = \frac{1}{\nu} = \frac{1}{500}$ s,
$$\lambda = \frac{v}{\nu} = \frac{350}{500} = \frac{7}{10}\ m$$
$\therefore$ $y = 0.2 \sin \left[1000\,\pi\,t - \frac{20\pi}{7} x\right]$
At $x = 3.5$ m and $t = 0.05$ s,
$$y = 0.2 \sin \left[1000\pi \times 0.05 - \frac{20\pi}{7} \times 3.5\right]$$
$$= 0.2 \sin (50\pi - 10\pi) = 0.2 \sin (40\pi) = \mathbf{0.}$$

7. Here $T = \frac{1}{\nu} = \frac{1}{75}$ s, $\lambda = \frac{v}{\nu} = \frac{45}{75} = 0.6$ m $= 60$ cm, $A = 2$ cm

$\therefore \quad y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$= 2 \sin 2\pi \left(75t - \frac{x}{60}\right)$

At $t = 3s$ and $x = 135$ cm,

$y = 2 \sin 2\pi (225 - 2.25)$

$= 2 \sin (450\pi - 4.5\pi)$

$= 2 \sin (-4.5\pi)$

$= -2 \sin (4.5\pi) = -2 \sin (4\pi + \pi/2)$

$= -2 \sin \pi/2 = $ **-2 cm.**

8. $\Delta\phi = \frac{2\pi}{\lambda} \Delta x \quad \therefore \quad \frac{2\pi}{3} = \frac{2\pi}{\lambda} \times 15$ or $\lambda =$ **45 cm.**

15.13 BOUNDARY EFFECTS

17. *Explain the phenomenon of reflection of waves by considering a wave pulse travelling along a string, whose one end is (i) fixed to a rigid support and (ii) tied to a ring which can freely slide up and down a vertical rod. What are the phase changes in each case ?*

Reflection of a wave from a rigid boundary. As shown in Fig. 15.11, consider a wave pulse travelling along a string (rarer medium) attached to a rigid support, such as a wall (denser medium). As the pulse reaches the wall, it exerts an upward force on the wall. By Newton's third law, the wall exerts an equal downward force on the string. This produces a reflected pulse in the downward direction, which travels in the reverse direction. Thus a crest is reflected as a trough.

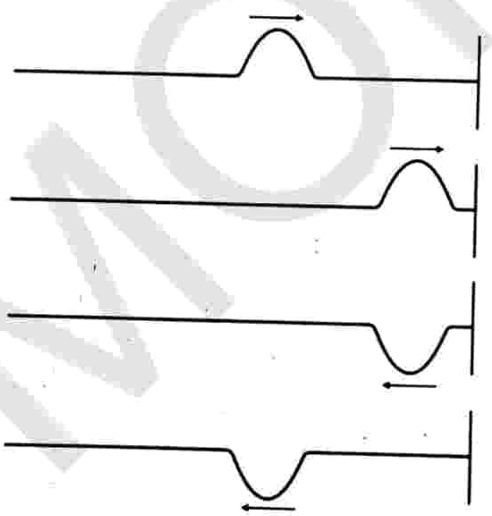


Fig. 15.11 Reflection of a pulse in a string from a rigid support.

Hence *when a travelling wave is reflected from a rigid boundary, it is reflected back with a phase reversal or phase difference of π radians.*

Reflection of a wave from an open boundary. As shown in Fig. 15.12, consider a wave pulse travelling along a string attached to a light ring, which slides without friction up and down a vertical rod. As the crest produced in the string at *A* reaches the end *B*, it meets little or no opposition there. The ring rises above its equilibrium position. As the ring moves up, it stretches the string and produces a reflected crest which travels back towards *A*. There is no phase reversal and crest is reflected as a crest.

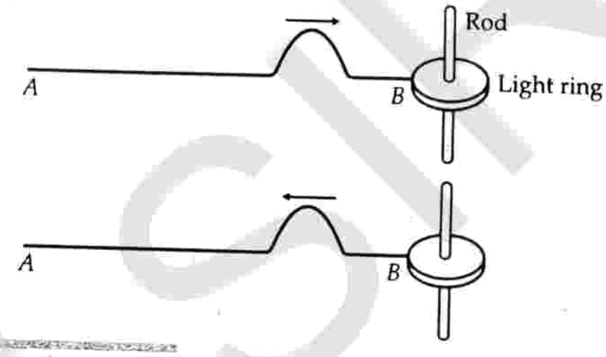


Fig. 15.12 Reflection of a wave pulse on a string from a free boundary.

Hence *when a travelling wave is reflected from a free or open boundary, it suffers no phase change.*

Suppose an incident wave is represented by

$$y_i(x, t) = A \sin(\omega t - kx)$$

For reflection at a rigid boundary, the reflected wave can be represented as

$$y_r(x, t) = -A \sin(\omega t + kx)$$

[Signs of both y and x change]

For reflection at an open boundary, the reflected wave can be represented as

$$y_r(x, t) = A \sin(\omega t + kx)$$

[Only sign of x changes]

Obviously, in case of reflection from a rigid boundary, the incident and reflected pulses meet in opposite phases at the end point and so cancel each other. Hence a ***node*** is formed at the boundary *i.e.*, the net displacement is zero.

In case of reflection from an open boundary, the incident and reflected pulses meet in same phase at the end point and reinforce (or get added to) each other. Hence an ***antinode*** is formed at the boundary *i.e.*, the displacement is maximum and is twice the amplitude of the either pulse.

17. *Considering the wave pulses travelling on stretched strings, discuss the phase change during the refraction of a wave.*

Refraction of a wave. Consider a combination of a thinner string *A* and a thicker string *B* kept under same tension. As $v = \sqrt{T/m}$, so a wavepulse travels faster on a thin string than that on a thick string *i.e.*, the thinner string *A* acts as rarer medium while the thicker string *B* acts as denser medium.

As shown in Fig. 15.13(*a*), when a wave pulse travels from thinner string to the thicker string, it is partly reflected and partly refracted at the interface. The reflected pulse travels faster while the refracted pulse travels slower. Also, the reflected pulse suffers a phase change of 180° while the refracted pulse does not suffer any phase change.

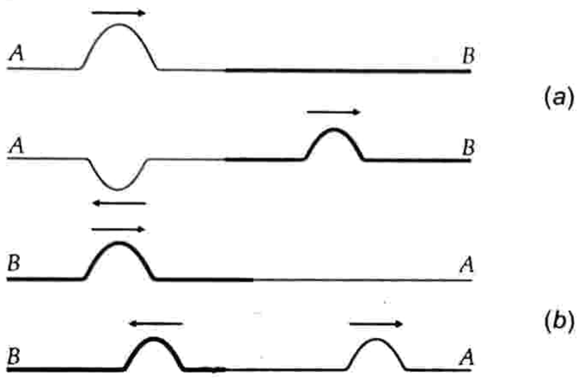


Fig. 15.13 Reflection and refraction of a pulse (*a*) when the second string is denser than the first and (*b*) when the second string is lighter than the first.

As shown in Fig. 15.13(*b*), when a pulse of crest travels from the thicker to the thinner string, both the transmitted and reflected pulses travel as crest *i.e.*, they do not suffer any phase change.

Hence *a wave suffers no phase change during its refraction from one medium to another.*

For Your Knowledge

- If a wave coming in a medium where its velocity is larger meets a medium where its velocity is smaller, it is reflected back with a reversal of phase or a phase change of π radians.
- If the wave initially comes from a medium where its velocity is smaller, the reflected wave does not suffer any phase change.
- The incident and reflected waves obey the usual laws of reflection. The frequency, wavelength and velocity of the reflected wave are same as those of incident wave.
- The wave transmitted into the second medium always goes without any change in phase.
- The incident and refracted rays obey the Snell's law of refraction.
- The wave velocity and wavelength of the refracted wave are different from those of the incident wave but their frequencies are equal. Hence $\nu = \frac{v_i}{\lambda_i} = \frac{v_r}{\lambda_r}$

  Here the subscripts *i* and *r* stand for the incident and the refracted waves respectively.

15.14 PRINCIPLE OF SUPERPOSITION OF WAVES

19. *What is meant by the independent behaviour of waves ?*

Independent behaviour of waves. A wave preserves its individuality while travelling through space. So *when a number of waves travel through a region at the same time, each wave travels independently of the others i.e.*, as if all other waves were absent. That is why, with so many different musical instruments playing simultaneously in a full orchestra, we can still identify the note produced by an individual instrument. An important consequence of the independent behaviour of the waves is the principle of superposition of waves.

20. *State and explain the principle of superposition of waves.*

Principle of superposition of waves. When one wave reaches a particle of the medium, the particle suffers one displacement. When two waves simultaneously cross this particle, it suffers two displacements, one due to each wave. *The resultant displacement of the particle is equal to the algebraic sum of the individual displacements given to it by the two waves. This is the principle of superposition of waves.*

The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves.

If $y_1, y_2, y_3, \ldots, y_n$ are the displacements due to waves acting separately, then according to the principle of superposition the resultant displacement, when all the waves act together is given by the algebraic sum

$$y = y_1 + y_2 + y_3 + \ldots + y_n$$
$$= f_1(vt - x) + f_2(vt - x) + \ldots + f_n(vt - x)$$
$$= \sum_{i=1}^{n} f_i(vt - x)$$

Explanation of the superposition principle. ***(i) Superposition of two identical pulses travelling towards each other.*** Fig. 15.14 shows two pulses moving towards each other with a speed of 1 ms^{-1}. The positions of the two pulses after every one second are shown in Figs. 15.14(*a*) to (*f*). They cross each other between $t = 2$ s and $t = 3$s. When the two pulses overlap or superpose, the displacement of the resultant pulse is twice the displacement of either pulse *i.e.*, equal to the sum of the displacements of the two pulses. This is the case of *constructive interference.*

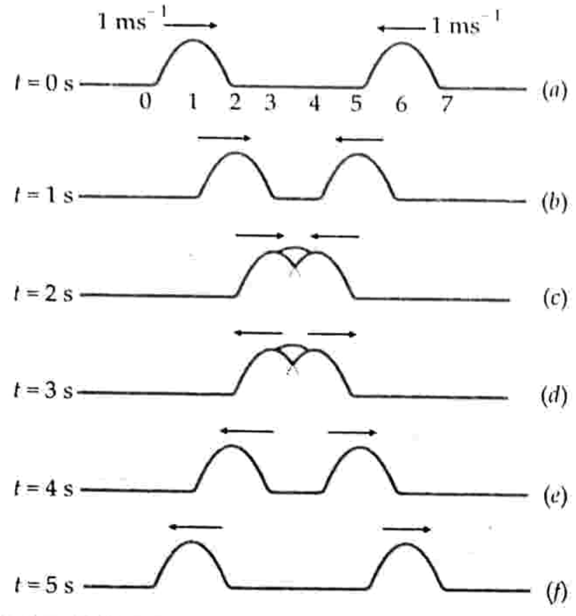


Fig. 15.14 Superposition of two identical pulses travelling in opposite directions.

(*ii*) ***Superposition of two pulses of equal and opposite shapes moving towards each other.*** Fig. 15.15 shows what happens when two equal and opposite pulses moving in opposite directions cross each other. In Fig. 15.15(*c*), we see that there is an instant when the string appears undisturbed. In this situation the positive pulse overlaps the negative pulse and seem to cancel each other. This is the case of *destructive interference*. This again shows that the resultant wave profile is the algebraic sum of individual waves.

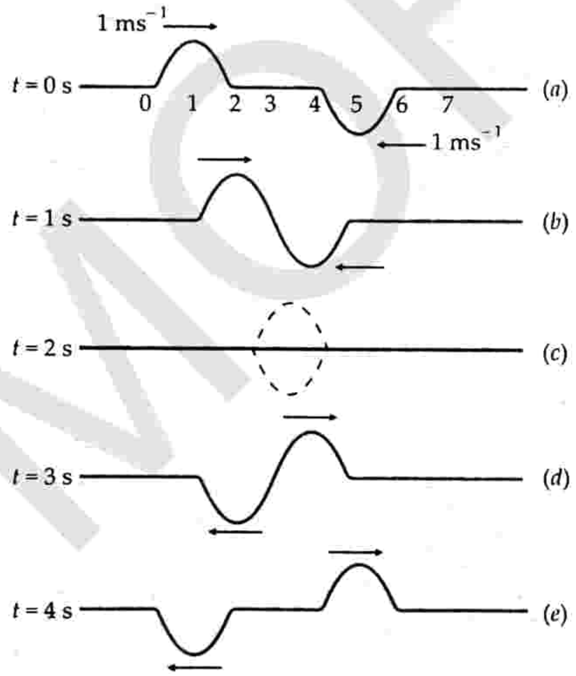


Fig. 15.15 Superposition of two equal and opposite pulses travelling in opposite directions.

Clearly, the two pulses continue to retain their individual shapes after crossing each other. However, at the instant they cross each other, the appearance of the wave profile is different from the shape of either individual pulse.

The superposition of *two* waves may lead to following *three* different effects :

(*i*) When two waves of the same frequency moving with the same speed in the same direction in a medium superpose on each other, they give rise to effect called ***interference of waves***.

(*ii*) When two waves of same frequency moving with the same speed in the opposite directions in a medium superpose on each other, they produce ***stationary waves***.

(*iii*) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each other, they produce ***beats***.

For Your Knowledge

- The principle of superposition holds not only for the mechanical waves but also for electromagnetic waves.
- In case of mechanical waves, the superposition principle does not hold if the amplitude of disturbance is so large that the ordinary linear laws of mechanical action no longer hold good. For example, the superposition principle fails in case of *shock waves* generated by a violent explosion.

15.15 STATIONARY WAVES

21. *What are stationary waves ? What is the necessary condition for the formation of stationary waves ? What are the two types of stationary waves ?*

Stationary waves. *When two identical waves of same amplitude and frequency travelling in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called stationary or standing wave.*

The resultant wave keeps on repeating itself in the same fixed position. Some particles of the medium remain permanently at rest *i.e.*, they have zero displacement. Their positions are called ***nodes***. Some other particles always suffer maximum displacement. Their positions are called ***antinodes***. The positions of nodes and antinodes do not change with time. That is why, such waves are called *stationary* or *standing waves*, to distinguish them from *progressive* or *travelling waves* which travel through the medium with a definite speed v. In such waves, there is no transfer of energy along the medium in either direction.

Necessary condition for the formation of stationary waves. A stationary wave cannot be formed from two independent waves travelling in a medium in opposite directions. In actual practice a stationary wave is produced when a progressive wave and its reflected wave are superposed. Hence a *stationary wave can be produced only in a finite medium which has its boundaries,* for example, a string of finite length or a rod, or a column of liquid or gas. The wave reflected from the boundary is of the same kind as the incident wave. The incident and reflected waves superpose each other continuously, giving rise to stationary waves.

Two types of stationary waves :

(*i*) ***Transverse stationary waves.*** *When two identical transverse waves travelling in opposite directions overlap, a transverse stationary wave is formed.* For example, transverse stationary waves are formed in a sonometer and Melde's experiment.

(*ii*) ***Longitudinal stationary waves.*** *When two identical longitudinal waves travelling in opposite directions overlap, a longitudinal stationary wave is formed.* For example, longitudinal stationary waves are formed in a resonance apparatus, organ pipes and Kundt's tube.

15.16 GRAPHICAL TREATMENT OF STATIONARY WAVES

22. *Explain graphically the formation of stationary waves and mark out clearly the positions of nodes and antinodes.*

Formation of stationary waves by graphical method. In Fig. 15.16, the full line curve represents a harmonic wave of time period T and wavelength λ travelling from left to right while the dashed curve represents an identical wave travelling from right to left. The resultant wave is obtained by taking the algebraic sum of the displacements of the two waves at every point and is shown by the thick line curve. The situations at different intervals of time are shown in Figs. 15.16(*i*) to (*v*).

(*i*) **At $t = 0$**, the two waves are in same phase *i.e.*, the crests and troughs of the two waves coincide respectively with each other. The amplitude of the resultant wave is twice of that due to each individual wave. All particles are at their positions of maximum displacement [Fig. 15.16(*i*)].

(*ii*) **At $t = T/4$**, each wave has advanced through a distance of $\lambda/4$ from the opposite direction. The two waves are in opposite phases. The resultant wave is the central straight line. All the particles of the medium are now passing through their mean positions [Fig. 15.16(*ii*)].

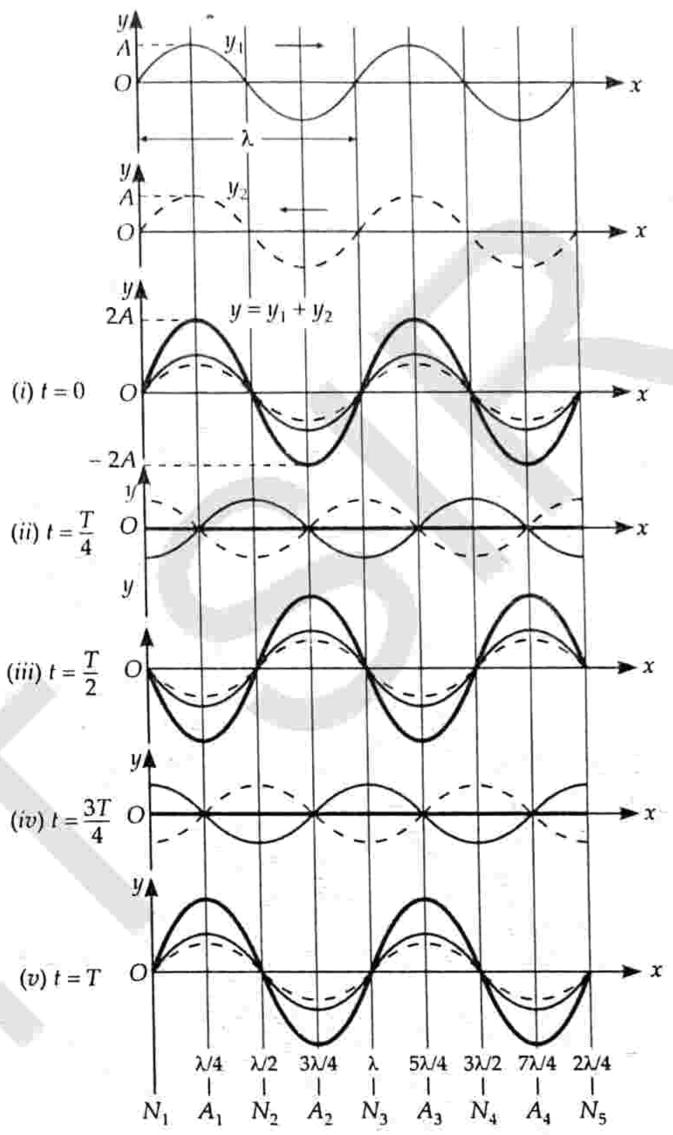


Fig. 15.16 Formation of stationary waves by graphical method.

(*iii*) **At $t = T/2$**, each wave has advanced through a distance of $\lambda/2$ from the opposite direction. The two waves are again in same phase. The resultant wave is reciprocal of that at $t = 0$. All the particles are at their positions of maximum displacement but in the directions opposite to those at $t = 0$ [Fig. 15.16(*iii*)].

(*iv*) **At $t = 3T/4$**, each wave has advanced through a distance of $3\lambda/4$ from the opposite direction. The two waves are again in opposite phases. The resultant wave is the central straight line. All the particles are again passing through their mean positions, but their directions of motion are opposite to those at $t = T/4$ [Fig. 15.16(*iv*)].

(*v*) **At $t = T$**, each wave has advanced through a distance λ from opposite direction. The two waves are again in same phase. The resultant wave is similar to that at $t = 0$. This completes one cycle [Fig. 15.16(*v*)].

The whole cycle continues to repeat again and again. The various segments of the string, on which stationary waves are formed, keep on vibrating up and down. The positions $N_1, N_2, N_3, ...$ where the amplitude of oscillation is zero are called ***nodes***. The positions $A_1, A_2, A_3, ...$ where the amplitude of oscillation is maximum are called ***antinodes***. Clearly, the separation between two successive nodes or antinodes is $\lambda/2$. The separation between a node and the next antinode is $\lambda/4$.

15.17 ANALYTICAL TREATMENT OF STATIONARY WAVES

23. *Obtain an expression for a stationary wave formed by two sinusoidal waves travelling along the same path in opposite directions and obtain the positions of nodes and antinodes.*

Analytical treatment of stationary waves. Consider two sinusoidal waves of equal amplitude and frequency travelling along a long string in opposite directions. The wave travelling along positive X-direction can be represented as

$$y_1 = A \sin(\omega t - kx)$$

The wave travelling along negative X-direction can be represented as

$$y_2 = A \sin(\omega t + kx)$$

According to the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2$$
$$= A \sin(\omega t - kx) + A \sin(\omega t + kx)$$
$$= 2A \sin \omega t \cos kx$$
$$[\because \sin(A+B) + \sin(A-B) = 2 \sin A \cos B]$$

or $$y = (2A \cos kx) \sin \omega t$$

This equation represents a stationary wave. It cannot represent a progressive wave because the argument of any of its trigonometric functions does not contain the combination $(\omega t \pm kx)$. The stationary wave has the same angular frequency ω but has amplitude

$$A' = 2A \cos kx$$

Obviously in case of a stationary wave, the amplitude of oscillation is not same for all the particles. It varies harmonically with the location x of the particle.

Changes with position x. The amplitude will be zero at points, where

$$\cos kx = 0$$

or $$kx = \left(n + \frac{1}{2}\right)\pi, \text{ where } n = 0, 1, 2, 3, ...$$

or $$\frac{2\pi x}{\lambda} = \left(n + \frac{1}{2}\right)\pi \qquad \left[\because k = \frac{2\pi}{\lambda}\right]$$

or $$x = (2n+1)\frac{\lambda}{4}$$

or $$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$$

These positions of zero amplitude are called ***nodes***. Clearly, the separation between two consecutive nodes is $\lambda/2$.

The amplitude will have a maximum value of $2A$ at points, where

$$\cos kx = \pm 1$$

or $$kx = n\pi, \text{ where } n = 0, 1, 2, 3, ...$$

or $$\frac{2\pi}{\lambda}x = n\pi \quad \text{or} \quad x = n\frac{\lambda}{2}$$

or $$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$$

These positions of maximum amplitude are called ***antinodes***. Clearly, the antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

Changes with time t. At the instants $t = 0, T/2, 3T/2, ...$, we have

$$\sin \omega t = \sin \frac{2\pi}{T} t = 0$$

Thus at these instants the displacement y becomes zero at all the points. That is, *all the particles of the medium pass through their mean positions simultaneously twice in each cycle.*

At the instants $t = T/4, 3T/4, 5T/4, ...$, we have

$$\sin \omega t = \sin \frac{2\pi}{T} t = \pm 1.$$

Thus at these instants the displacement y is maximum at all the points and becomes alternately positive and negative. That is, *all the particles of the medium pass through their positions of maximum displacements twice in each cycle.*

15.18 CHARACTERISTICS OF STATIONARY WAVES

24. *Mention some of the important characteristics of stationary waves.*

Characteristics of stationary waves :

(*i*) In a stationary wave, the disturbance does not advance forward. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite conditions after every half the time period.

(*ii*) All particles of the medium, except those at nodes, execute simple harmonic motions with the same time period about their mean positions.

(*iii*) During the formation of a stationary wave, the medium is broken into loops or segments between equally spaced points called *nodes* which remain permanently at rest and midway between them are points called *antinodes* where the displacement amplitude is maximum.

(*iv*) The distance between two successive nodes or antinodes is $\lambda/2$.

(*v*) The amplitudes of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

(*vi*) The maximum velocity is different at different points. Its value is zero at the nodes and progressively increases towards the antinode. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.

(*vii*) All the particles in a particular segment between two nodes vibrate in the same phase but the particles in two neighbouring segments vibrate in opposite phases, as shown in Fig. 15.17.

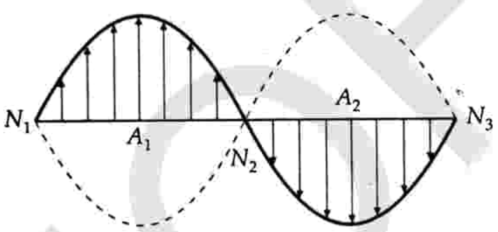


Fig. 15.17 Opposite phases of particles in consecutive segments.

(*viii*) Twice in each cycle, the energy becomes alternately wholly potential and wholly kinetic. It is wholly kinetic when the particles are at their positions of maximum displacements and wholly kinetic when the particles pass through their mean positions.

(*ix*) There is no transference of energy across any section of the medium because no energy can flow past a nodal point which remains permanently at rest.

(*x*) A stationary wave has the same wavelength and time period as the two component waves.

15.19 COMPARISON BETWEEN STATIONARY AND PROGRESSIVE WAVES

25. *Give important differences between progressive and stationary waves.*

Progressive waves	***Stationary waves***
1. The disturbance travels forward with a definite velocity.	The disturbance remains confined to the region where it is produced.
2. Each particle of the medium executes SHM about its mean position with the same amplitude.	Except nodes, all particles of the medium execute SHM with varying amplitude.
3. There is a continuous change of phase from one particle to the next.	All the particles between two successive nodes vibrate in the same phase, but the phase reverses for particles between next pair of nodes.
4. No particle of the medium is permanently at rest.	The particles of the medium at nodes are permanently at rest.
5. There is no instant when all the particles are at the mean positions together.	Twice during each cycle, all particles pass through their mean positions simultaneously.
6. There is flow of energy across every plane along the direction of propagation of the wave.	Energy of one region remains confined in that region.
7. The energy averaged over a wavelength is half kinetic and half potential.	Twice during each cycle, the energy becomes alternately wholly potential and wholly kinetic.
8. All the particles have same maximum velocity which they attain while passing their mean position one after the other.	All the particles attain their individual maximum velocities at the same time as they pass through their mean positions. This velocity varies from zero at nodes to maximum at antinodes.

Examples based on Equation of Stationary Waves

FORMULAE USED

1. Let $y_1 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (incident wave)

$y_2 = \pm\, a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$ (reflected wave)

Then stationary wave formed by the superposition is given by

$$y = y_1 + y_2 = \pm\, 2a \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi t}{T}$$

It involves the product of separate harmonic functions of time t and position x.

2. For (+) sign in the above equation, antinodes are formed at the positions $x = 0,\ \lambda/2,\ \lambda,\ 3\lambda/2, \ldots$ and nodes are formed at $x = \lambda/4,\ 3\lambda/4,\ 5\lambda/4, \ldots$
3. For (−) sign, antinodes are formed at the positions $x = \lambda/4,\ 3\lambda/4,\ 5\lambda/4, \ldots$ and nodes at $x = 0,\ \lambda/2,\ \lambda,\ 3\lambda/2, \ldots$
4. The distance between two successive nodes or antinodes is $\lambda/2$ and that between a node and nearest antinode is $\lambda/4$.

UNITS USED

Units of y and a are same cm or m, units of x and λ are same cm or m.

EXAMPLE 32. *The constituent waves of a stationary wave have amplitude, frequency and velocity as 8 cm, 30 Hz and 180 cm s^{-1} respectively. Write down the equation of the stationary wave.*

Solution. Here $a = 8$ cm, $\nu = 30$ Hz,

$$v = 180 \text{ cm s}^{-1},\quad T = 1/\nu = 1/30 \text{ s}$$

$$\lambda = \frac{v}{\nu} = \frac{180}{30} = 6 \text{ cm}$$

Equation of stationary wave is

$$y = 2a \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi t}{T}$$

$$= 2 \times 8 \cos\frac{2\pi x}{6} \sin\frac{2\pi t}{1/30}$$

or $$\mathbf{y = 16 \cos\frac{\pi x}{3} \sin 60\pi t.}$$

EXAMPLE 33. *Stationary waves are set up by the superposition of two waves given by*

$$y_1 = 0.05 \sin(5\pi t - x) \text{ and } y_2 = 0.05 \sin(5\pi t + x)$$

where x and y are in metres and t in seconds. Find the displacement of a particle situated at a distance $x = 1$ m.

Solution. According to principle of superposition, the resultant displacement is given by

$$y = y_1 + y_2$$

$$= 0.05 \sin(5\pi t - x) + 0.05 \sin(5\pi t + x)$$

$$= 0.05 \times 2 \sin\frac{5\pi t - x + 5\pi t + x}{2} \cos\frac{5\pi t + x - 5\pi t + x}{2}$$

$$= 0.1 \cos x \sin 5\pi t$$

∴ Amplitude, $A = 0.1 \cos x$

At $x = 1$ m,

$$A = 0.1 \cos 1 = 0.1 \cos\frac{180°}{\pi} = 0.1 \cos\frac{180°}{3.14}$$

$$= 0.1 \cos 57.3° = 0.1 \times 0.5402 = \mathbf{0.054\ m.}$$

PROBLEMS FOR PRACTICE

1. The distance between two consecutive nodes in a stationary wave is 25 cm. If the speed of the wave is 300 ms^{-1}, calculate the frequency. (**Ans.** 600 Hz)
2. The equation of a longitudinal stationary wave produced in a closed organ pipe is

$$y = 6 \sin\frac{2\pi x}{6} \cos 160\pi t$$

where x, y are in cm and t in seconds. Find (*i*) the frequency, amplitude and wavelength of the original progressive wave (*ii*) separation between two successive nodes and (*iii*) equation of the original progressive wave.

[**Ans.** (*i*) $\nu = 80$ Hz, $a = 3$ cm, $\lambda = 6$ cm (*ii*) 3 cm (*iii*) $y = 3 \sin\left(\frac{\pi}{3}x - 160\pi t\right)$]

3. (*i*) Write the equation of a wave identical to the wave represented by the equation :

$$y = 5 \sin \pi(4.0t - 0.02x)$$

but moving in opposite direction.

(*ii*) Write the equation of stationary wave produced by the composition of the above two waves and determine the distance between two nearest nodes. All the distances in the equation are in mm.

[**Ans.** (*i*) $y = 5 \sin \pi(4.0t + 0.02x)$ (*ii*) $y = 10 \cos 0.02\pi x \sin 4.0\pi t$, 50 mm]

15.20 STATIONARY WAVES IN A STRING FIXED AT BOTH ENDS

26. *Give a qualitative discussion of the modes of vibrations of a stretched string fixed at both the ends.*

Normal modes of vibration of a stretched string : Qualitative discussion. Consider a string clamped to rigid supports at its ends. If the wire be plucked in the middle, traverse waves travel along it and get reflected from the ends. These identical waves travelling in opposite directions give rise to stationary waves. Due to boundary conditions, the string vibrates in one or more segments or loops with certain natural frequencies. These special patterns are called *normal modes*.

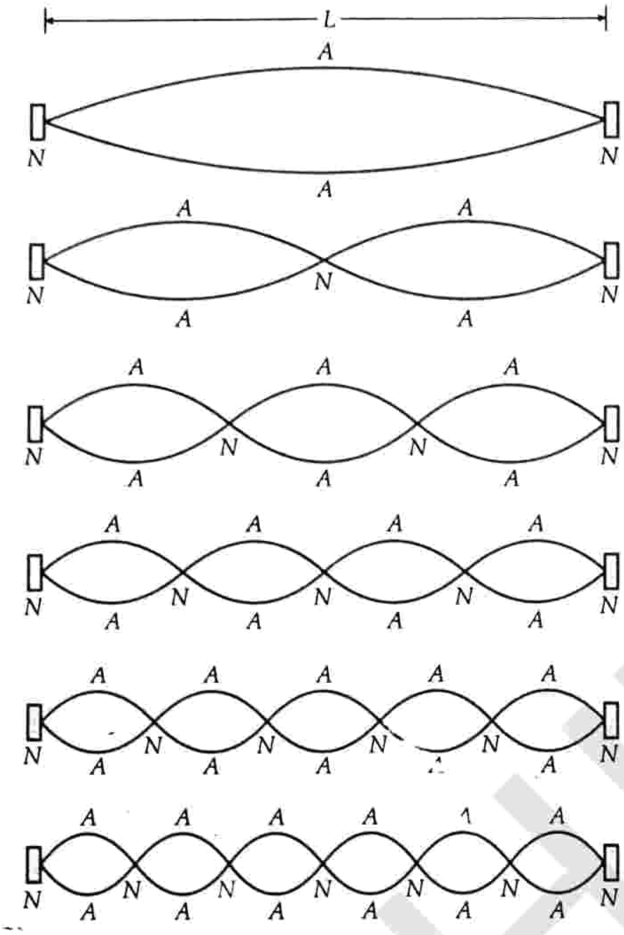


Fig. 15.18 Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

Consider a string of length L, stretched under tension T. Let m be the mass per unit length of string. The speed of the transverse wave on the string will be

$$v = \sqrt{\frac{T}{m}}$$

As the two ends of the string are fixed, they remain at rest. So there is a node N at each end. The different modes of vibration of stretched string fixed at both the ends are shown in Fig. 15.18.

First mode of vibration. If the string is plucked in the middle and released, it vibrates in one segment with nodes at its ends and an antinode in the middle.

Here length of string,

$$L = \frac{\lambda_1}{2} \quad \text{or} \quad \lambda_1 = 2L$$

$\therefore$ Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \text{ (say)}$$

This is the minimum frequency with which the string can vibrate and is called *fundamental note* or *first harmonic*

Second mode of vibration. If the string is pressed in the middle and plucked at one-fourth length, then the string vibrates in two segments.

Here $\quad L = 2 \cdot \frac{\lambda_2}{2} \quad$ or $\quad \lambda_2 = L$

$\therefore$ Frequency of vibration,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{T}{m}} = 2\nu$$

This frequency is called *first overtone* or *second harmonic.*

Third mode of vibration. If string is pressed at one-third of its length from one end and plucked at one-sixth length, it will vibrate in three segments. Then

$$L = 3\,\frac{\lambda_3}{2} \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

$\therefore$ Frequency of vibration,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu$$

This frequency is called *second overtone* or *third harmonic*. In general, if the string vibrates in p segments, then

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu.$$

27. *Give an analytical treatment of stationary waves in a stretched string.*

Analytical treatment of stationary waves in a string fixed at both the ends. Consider a uniform string of length L stretched by a tension T along the x-axis, with its ends rigidly fixed at the end $x = 0$ and $x = L$. Suppose a transverse wave produced in the string travels along the string along positive x-direction and gets reflected at the fixed end $x = L$. The two waves can be represented as

$$y_1 = A \sin(\omega t - kx)$$

and $$y_2 = -A \sin(\omega t + kx)$$

The negative sign before A is due to phase reversal of the reflected wave at the fixed end. By the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2 = -A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$

$$= -2A \cos \omega t \sin kx$$

$$[\sin(A+B) - \sin(A-B) = 2 \cos A \sin B]$$

or $$y = -2A \sin kx \cos \omega t \qquad \text{...(1)}$$

If stationary waves are formed, then the ends $x = 0$ and $x = L$ must be nodes because they are kept fixed. So, we have the boundary conditions :

$$y = 0 \quad \text{at } x = 0 \quad \text{for all } t$$

and $$y = 0 \quad \text{at } x = L \quad \text{for all } t$$

The first boundary condition $(y = 0, x = 0)$ is satisfied automatically by equation (1). The second boundary condition $(y = 0, x = L)$ will be satisfied if

$$y = -2 \sin kL \cos \omega t = 0$$

This will be true for all values of t only if

$$\sin kL = 0 \quad \text{or} \quad kL = n\pi, \text{ where } n = 1, 2, 3, \ldots$$

or $$\frac{2\pi L}{\lambda} = n\pi$$

For each value of n, there is a corresponding value of λ, so we can write

$$\frac{2\pi L}{\lambda_n} = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

The speed of transverse wave on a string of linear mass density m is given by

$$v = \sqrt{\frac{T}{m}}$$

So the frequency of vibration of the string is

$$\nu_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For $n = 1$, $$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \quad \text{(say)}$$

This is the lowest frequency with which the string can vibrate and is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $$\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu$$

(*First overtone or second harmonic*)

For $n = 3$, $$\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu$$

(*Second overtone or third harmonic*)

For $n = 4$, $$\nu_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4\nu$$

(*Third overtone or fourth harmonic*)

Thus the various frequencies are in the ratio 1 : 2 : 3 : ... and hence form a harmonic series. These frequencies are called ***harmonics*** with the fundamental itself as the *first harmonic*. The higher harmonic are called ***overtones***. Thus second harmonic is first overtone, third harmonic is second overtone and so on. These are shown in Fig. 15.18.

Nodes. *These are the positions of zero amplitude.* In the n^{th} mode of vibration, there are $(n + 1)$ nodes, which are located from one end at distances

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \ldots\ldots, L$$

Antinodes. *These are positions of maximum amplitude.* In the n^{th} mode of vibration, there are n antinodes, which are located at distances

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots\ldots, \frac{(2n-1)L}{2n}.$$

28. *State the laws of vibrations of stretched strings.*

Laws of transverse vibrations of a string. The fundamental frequency produced in a stretched string of length L under tension T and having mass per unit length m is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

The above equation gives the following laws of vibrations of strings :

(*i*) **Law of length.** *The fundamental frequency of a vibrating string is inversely proportional to its length provided its tension and mass per unit length remain the same.*

$$\nu \propto \frac{1}{L} \qquad (T \text{ and } m \text{ are constants})$$

(*ii*) **Law of tension.** *The fundamental frequency of a vibrating string is proportional to the square root of its tension provided its length and the mass per unit length remain the same.*

$$\nu \propto \sqrt{T} \qquad (L \text{ and } m \text{ are constants})$$

(*iii*) **Law of mass.** *The fundamental frequency of a vibrating string is inversely proportional to the square root of its mass per unit length provided the length and tension remain the same.*

$$\nu \propto \frac{1}{\sqrt{m}} \qquad (L \text{ and } T \text{ are constants})$$

If ρ is the density of the string and D its diameter, then its mass per unit length will be

$$m = \text{Volume of unit length} \times \text{density}$$

$$= \pi\left(\frac{D^2}{4}\right)\rho$$

$$\therefore \quad \nu = \frac{1}{2L}\sqrt{\frac{T}{\pi D^2 \rho / 4}}$$

$$= \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}.$$

Hence the law of mass may be expressed in the form of following two alternative laws :

(*a*) **Law of diameter.** *The fundamental frequency of a vibrating string is inversely proportional to its diameter provided its tension, length and density remain the same.*

$$\nu \propto \frac{1}{D} \qquad (T, L \text{ and } \rho \text{ are constants})$$

(*b*) **Law of density.** *The fundamental frequency of a vibrating string is inversely proportional to the square root of its density provided its tension, length and diameter remain constant.*

$$\nu \propto \frac{1}{\sqrt{\rho}} \qquad (T, L \text{ and } D \text{ are constants})$$

Examples based on Modes of Vibrations of Strings

FORMULAE USED

1. Fundamental frequency, $\nu = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}}$
2. When the stretched string vibrates in p loops,
$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu$$
3. For a string of diameter D and density ρ,
$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$
4. Law of length, $\nu \propto 1/L$ or $\nu L =$ constant
or $\nu_1 L_1 = \nu_2 L_2$.

UNITS USED

Tension T is in newton, linear mass density m in N kg^{-1}, length L in metre, density ρ in kg m^{-3}, frequency ν in Hz.

EXAMPLE 34. *A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg wt into between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a parmanent magnet and it vibrates in resonance, when carrying an alternating current of frequency ν. Find the frequency of the alternating source.* **[AIEEE 03]**

Solution. Here $m = 9.8 \text{ gm}^{-1} = 9.8 \times 10^{-3} \text{ kg m}^{-1}$,

$L = 1$ m; $T = 10$ kg wt $= 10 \times 9.8$ N $= 98$ N

Frequency of vibration of the string

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 1}\sqrt{\frac{98}{9.8 \times 10^{-3}}} = 50 \text{ Hz}$$

As the string vibrates in resonance, the frequency of alternating current, $n =$ **50 Hz.**

EXAMPLE 35. *Calculate the fundamental frequency of a sonometer wire of length = 20 cm, tension 25 N, cross-sectional area $= 10^{-2}$ cm^2 and density of the material of wire $= 10^4$ kg m^{-3}.*

Solution. Here $L = 20 \text{ cm} = 0.20 \text{ m}$, $T = 25$ N,

$A = 10^{-2} \text{ cm}^2 = 10^{-6} \text{ m}^2$, $\rho = 10^4 \text{ kg m}^{-3}$

Mass per unit length of the wire,

$$m = A \times 1 \times \rho = 10^{-6} \times 10^4 \text{ kg m}^{-1} = 10^{-2} \text{ kg m}^{-1}$$

Fundamental frequency of the sonometer wire is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.20}\sqrt{\frac{25}{10^{-2}}} = \frac{50}{2 \times 0.20} = \mathbf{125 \text{ Hz.}}$$

EXAMPLE 36. *The length of a sonometer wire is 0.75 m and density 9×10^3 kg m^{-3}. It can bear a stress of 8.1×10^8 N/m^2 without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire ?* **[IIT 90]**

Solution. Here $L = 0.75$ m, $\rho = 9 \times 10^3$ kg m^{-3},

Stress $= 8.1 \times 10^9$ Nm^{-2}

Let a be the area of cross-section of the wire, then fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{\text{Stress} \times a}{a \times 1 \times \rho}} = \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\rho}}$$

$$= \frac{1}{2 \times 0.75}\sqrt{\frac{8.1 \times 10^8}{9 \times 10^3}} = \mathbf{200 \text{ Hz.}}$$

EXAMPLE 37. *A stretched wire emits a fundamental note of 256 Hz. Keeping the stretching force constant and reducing the length of wire by 10 cm, the frequency becomes 320 Hz. Calculate the original length of the wire.*

Solution. Frequency of fundamental note,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

In first case : $256 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

In second case : $320 = \frac{1}{2(L-10)}\sqrt{\frac{T}{m}}$

On dividing, we get

$$\frac{320}{256} = \frac{2L}{2(L-10)} \quad \text{or} \quad \frac{L}{L-10} = \frac{5}{4}$$

or $L =$ **50 cm.**

EXAMPLE 38. *Find the fundamental note emitted by a string of length $10\sqrt{10}$ cm under tension of 31.4 kg. Radius of string is 0.55 mm and density = 9.8 g cm^{-3}.*

Solution. Here $L = 10\sqrt{10}$ cm $= 0.1\sqrt{10}$ m,

$T = 3.14$ kg wt $= 3.14 \times 9.8$ N, $r = 0.5$ mm

or $D = 1$ mm $= 10^{-3}$ m,

$\rho = 9.8$ g cm^{-3} $= 9800$ kg m^{-3}

Fundamental note,

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

$$= \frac{1}{0.1\sqrt{10} \times 10^{-3}}\sqrt{\frac{3.14 \times 9.8}{3.14 \times 9800}} = \mathbf{100\ Hz.}$$

EXAMPLE 39. *A rope 5 m long has a total mass of 245 g. It is stretched with a constant tension of 1 kg wt. If it is fixed at one end and shaken by hand at the other end, what frequency of shaking will make it break up into three vibrating segments ? Take $g = 980\ cm\,s^{-2}$.*

Solution. Here $L = 5$ m $= 500$ cm,

$T = 1$ kg wt $= 1000 \times 980$ dyne, $p = 3$

$$m = \frac{245\text{ g}}{500\text{ cm}} = 0.49\text{ g cm}^{-1}$$

When the rope vibrates in p segments, its frequency of vibration is

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad \nu_3 = \frac{3}{2 \times 500}\sqrt{\frac{1000 \times 980}{0.49}} = \mathbf{4.24\ Hz.}$$

EXAMPLE 40. *In an experiment it was found that the string vibrated in three loops when 8 g were placed on the scale pan. What mass must be placed on the pan to make the string vibrate in six loops ? Neglect the mass of the string and the scale pan.*

Solution. Frequency of vibration of a string vibrating in p loops is

$$\nu = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

As the quantities ν, L and m remain constant, so

$T \times p^2 =$ constant or $T_1 \times p_1^2 = T_2 \times p_2^2$

or $$T_2 = \frac{p_1^2}{p_2^2} \times T_1$$

Given : $p_1 = 3$, $T_1 = 8$ g, $p_2 = 6$

$$\therefore \quad T_2 = \left(\frac{3}{6}\right)^2 \times 8 = \mathbf{2\ g.}$$

EXAMPLE 41. *A wire of length 108 cm produces a fundamental note of frequency 256 Hz, when stretched by a weight of 1 kg. By how much its length should be increased so that its pitch is raised by a major tone, if it is now stretched by a weight of 4 kg ?*

Solution. *In first case.*

$\nu = 256$ Hz, $L = 108$ cm, $T = 1 \times 1000 \times 980$ dyne

As $$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 256 = \frac{1}{2 \times 108}\sqrt{\frac{1 \times 1000 \times 980}{m}} \quad \text{...(1)}$$

In second case. Major tone,

$$\nu' = \frac{9}{8}\nu = \frac{9}{8} \times 256 = 288\text{ Hz,}$$

$$T = 4 \times 1000 \times 980\text{ dyne}$$

Let the length of the wire be increased by x cm. Its new length will be

$$L' = L + x = (108 + x)\text{ cm}$$

Now $$\nu' = \frac{1}{2L'}\sqrt{\frac{T'}{m}}$$

or $$288 = \frac{1}{2 \times (108 + x)}\sqrt{\frac{4 \times 1000 \times 980}{m}} \quad \text{...(2)}$$

Dividing (1) by (2), we get

$$\frac{256}{288} = \frac{108 + x}{108} \times \frac{1}{\sqrt{4}}$$

$$\frac{8}{9} = \frac{108 + x}{216}$$

On solving, x = **84 cm.**

EXAMPLE 42. *The length of a wire between the two ends of a sonometer is 105 cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15 ?*

Solution. Total length of the wire,

$$L = 105\text{ cm}$$

$$\nu_1 : \nu_2 : \nu_3 = 1 : 3 : 15$$

Let L_1, L_2 and L_3 be the lengths of the three parts.

As $$\nu \propto \frac{1}{L}$$

$$\therefore \quad L_1 : L_2 : L_3 = \frac{1}{1} : \frac{1}{3} : \frac{1}{15} = 15 : 5 : 1$$

Sum of the ratios $= 15 + 5 + 1 = 21$

$$\therefore \quad L_1 = \frac{15}{21} \times 105 = 75\text{ cm ;}$$

$$L_2 = \frac{5}{21} \times 105 = 25\text{ cm ;}$$

$$L_3 = \frac{1}{21} \times 105 = 5\text{ cm}$$

Hence the bridges should be placed at **75 cm** and (75 + 25 =)**100 cm** from one end.

EXAMPLE 43. The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. How will the frequency be affected if its length is increased by 10% ?

Solution. Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \quad ...(1)$$

On increasing the tension by 21%, the new tension becomes 1.21 T. Therefore,

$$\nu + 5 = \frac{1}{2L}\sqrt{\frac{1.21\,T}{m}} \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{\nu+5}{\nu} = \sqrt{1.21} = 1.1 \quad \text{or} \quad \nu = 50 \text{ Hz}$$

On increasing the length by 10%, the new frequency becomes

$$\nu' = \frac{1}{2\times 1.1L}\sqrt{\frac{T}{m}} = \frac{\nu}{1.1} = \frac{50}{1.1} = \mathbf{45.45\ Hz.}$$

EXAMPLE 44. A stone hangs in air from one end of a wire which is stretched over a sonometer. The wire is in unison with a certain tuning fork when the bridges of the sonometer are 45 cm apart. Now the stone hangs immersed in water at 4°C and the distance between the bridges has to be altered by 9 cm to re-establish unison of the wire with the same fork. Calculate the density of the stone.

Solution. Let V be the volume and ρ be the density of the stone. When the stone hangs in air, tension in the string is

$$T = V\rho g$$

Frequency of vibration,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{V\rho g}{m}} \quad ...(1)$$

When the stone is immersed in water, it loses in weight due to the upthrust of water. Tension in the string decreases to T'.

$$T' = \text{Apparent weight of stone}$$
$$= V\rho g - V\sigma g = V(\rho - \sigma) g$$

where σ is the density of water. The length of the wire has to be decreased to L' to bring it in unison with the tuning fork.

$$\therefore \quad \nu = \frac{1}{2L'}\sqrt{\frac{T'}{m}} = \frac{1}{2L'}\sqrt{\frac{V(\rho-\sigma)g}{m}} \quad ...(2)$$

From equations (1) and (2), we get

$$\frac{1}{2L}\sqrt{\frac{V\rho g}{m}} = \frac{1}{2L'}\sqrt{\frac{V(\rho-\sigma)g}{m}}$$

or

$$\frac{L'}{L} = \sqrt{\frac{\rho-\sigma}{\rho}}$$

Now $L = 45$ cm, $L' = 45 - 9 = 36$ cm, $\sigma = 1 \text{ g m}^{-3}$

$$\therefore \quad \frac{36}{45} = \sqrt{\frac{\rho - 1}{\rho}}$$

$$\frac{4}{5} = \sqrt{\frac{\rho-1}{\rho}}$$

or

$$\frac{\rho-1}{\rho} = \frac{16}{25} \quad \text{or} \quad 25\rho - 25 = 16\rho$$

$$\therefore \quad \rho = 25/9 = \mathbf{2.778\ g\ cm^{-3}}.$$

EXAMPLE 45. A wire having a linear mass density of 5.0×10^{-3} kg m^{-1} is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire. **[IIT]**

Solution. Suppose the wire resonates at 420 Hz in its pth harmonic and at 490 Hz in its $(p+1)$th harmonic.

As
$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 420 = \frac{p}{2L}\sqrt{\frac{T}{m}} \quad ...(1)$$

and

$$490 = \frac{p+1}{2L}\sqrt{\frac{T}{m}} \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{490}{420} = \frac{p+1}{p}$$

or

$$1 + \frac{70}{420} = 1 + \frac{1}{p}$$

or

$$p = 6$$

Putting the value of p in (1), we get

$$420 = \frac{6}{2L}\sqrt{\frac{450}{5\times 10^{-3}}} = \frac{3}{L}\sqrt{9\times 10^4}$$

$$[\because m = 5.0\times 10^{-3} \text{ kg m}^{-1}, T = 450 \text{ N}]$$

or

$$L = \frac{3\times 3\times 10^2}{420} = \frac{15}{7} = \mathbf{2.14\ m.}$$

PROBLEMS FOR PRACTICE

1. A sonometer wire is under a tension of 40 N and the length between the bridges is 50 cm. A metre long wire of the sonometer has a mass of 1.0 g. Determine its fundamental frequency. (**Ans.** 200 Hz)
2. A cord 80 cm long is stretched by a load of 8.0 kg f. The mass per unit length of the cord is 4.0×10^{-5} kg m^{-1}. Find (*i*) speed of the transverse wave in the cord and (*ii*) frequency of the fundamental and that of the second overtone.

   [**Ans.** (*i*) 1400 ms^{-1} (*ii*) 875 Hz, 2625 Hz]

3. The length of a stretched wire is 1 m and its fundamental frequency is 300 Hz. What is the speed of the transverse wave in the wire ? (**Ans.** 600 ms^{-1})

4. The mass of a 1 m long steel wire is 20 g. The wire is stretched under a tension of 800 N. What are the frequencies of its fundamental mode of vibration and the next three higher modes ? **[Roorkee 82]** (**Ans.** 100, 200, 300, 400 Hz)

5. If the tension in the string is increased by 5 kg wt, the frequency of the fundamental tone increases in the ratio 2 : 3. What was the initial tension in the string ? (**Ans.** 4 kg wt)

6. A sonometer wire has a length of 114 cm between its two fixed ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ? **[Roorkee 84]** (**Ans.** At a distance of 72 cm, 96 cm from one end)

7. Two wires of the same material are stretched with the same force. Their diameters are 1.2 mm and 1.6 mm, while their lengths are 90 cm and 60 cm respectively. If the frequency of vibrations of first is 256 Hz, find that of the other. (**Ans.** 288 Hz)

8. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz ? (**Ans.** 60 cm)

9. The ratio of frequencies of two wires having same length and same tension and made of the same material is 2 : 3. If the diameter of one wire be 0.09 cm, then determine the diameter of the other. (**Ans.** 0.06 cm)

10. A 50 cm long wire is in unison with a tuning fork of frequency 256, when stretched by a load of density 9 g cm^{-3} hanging vertically. The load is then immersed in water. By how much the length of the wire be reduced to bring it again in unison with the same tuning fork ? (**Ans.** 2.867 cm)

11. A string vibrates with a frequency of 200 Hz. Its length is doubled and its tension is altered until it begins to vibrate with a frequency of 300 Hz. What is the ratio of new tension to the original tension ? (**Ans.** 9 : 1)

12. In Melde's experiment, a string vibrates in 3 loops when 8 grams were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops ? (**Ans.** 2.88 g)

HINTS

2. (*i*) $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{8 \times 9.8}{4 \times 10^{-5}}} = \mathbf{1400\ ms^{-1}}.$

(*ii*) Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.80} \times 1400 = \mathbf{875\ Hz.}$$

Second overtone or third harmonic,

$$\nu_3 = 3\nu = 3 \times 875 = \mathbf{2625\ Hz.}$$

3. Fundamental frequency, $\nu = \frac{v}{2L}$

$\therefore \quad v = 2L\nu = 2 \times 1 \times 300 = \mathbf{600\ ms^{-1}}.$

5. $\sqrt{\frac{T_1}{T_2}} = \frac{\nu_1}{\nu_2} \quad \therefore \quad \frac{T_1}{T_1 + 5} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

or $\quad 9T_1 = 4T_1 + 20$

or $\quad T_1 = \mathbf{4\ kg\ wt.}$

7. $\nu_1 = \frac{1}{L_1 D_1}\sqrt{\frac{T}{\pi\rho}}, \quad \nu_2 = \frac{1}{L_2 D_2}\sqrt{\frac{T}{\pi\rho}}$

$\therefore \quad \frac{\nu_2}{\nu_1} = \frac{L_1 D_1}{L_2 D_2} = \frac{90}{60} \times \frac{1.2}{1.6} = \frac{9}{8};$

$$\nu_2 = \frac{9}{8}\nu_1 = \frac{9}{8} \times 256 = \mathbf{288\ Hz.}$$

8. Here $L_1 = 90$ cm, $\nu_1 = 124$ Hz, $\nu_2 = 186$ Hz, $L_2 = ?$

According to the law of length,

$$\nu_2 L_2 = \nu_1 L_1$$

$\therefore \quad L_2 = \frac{\nu_1 L_1}{\nu_2} = \frac{124 \times 90}{186} = \mathbf{60\ cm.}$

10. As proved in Example 44,

$$\frac{L'}{L} = \sqrt{\frac{\rho - \sigma}{\rho}}$$

Here $L = 50$ cm, $\rho = 9\ g\ cm^{-3}$, $\sigma = 1\ g\ cm^{-3}$

$\therefore \quad \frac{L'}{50} = \sqrt{\frac{9-1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} = \frac{2 \times 1.414}{3}$

or $\quad L' = \frac{141.4}{3} = 47.133$ cm

Decrease in length $= L - L' = 50 - 47.133 = \mathbf{2.867\ cm.}$

11. In first case, $\quad 200 = \frac{1}{2L}\sqrt{\frac{T_1}{m}} \quad$ ...(1)

In second case, $\quad 300 = \frac{1}{2 \times 2L}\sqrt{\frac{T_2}{m}} \quad$ ...(2)

Dividing (2) by (1), we get

$$\frac{300}{200} = \frac{1}{2}\sqrt{\frac{T_2}{T_1}} \quad \text{or} \quad \frac{3}{1} = \sqrt{\frac{T_2}{T_1}}$$

or $\quad \frac{T_2}{T_1} = \frac{9}{1} = \mathbf{9 : 1.}$

12. In first case, $\nu = \frac{3}{2L}\sqrt{\frac{T_1}{m}}$

In second case, $\nu = \frac{5}{2L}\sqrt{\frac{T_2}{m}}$

$$\therefore \frac{3}{2L}\sqrt{\frac{T_1}{m}} = \frac{5}{2L}\sqrt{\frac{T_2}{m}}$$

or $\sqrt{\frac{T_2}{T_1}} = \frac{3}{5}$ or $\frac{T_2}{T_1} = \frac{9}{25}$

$$T_2 = \frac{9}{25} \times T_1 = \frac{9}{25} \times 8 = \mathbf{2.88\ g\ wt.}$$

15.21 STATIONARY WAVES IN ORGAN PIPES OR AIR COLUMNS

29. *What is an organ pipe ? How does it produce sound ?*

Organ pipe. *It is the simplest musical instrument in which sound is produced by setting an air column into vibrations.* Fig. 15.19 shows the section of a flute organ pipe.

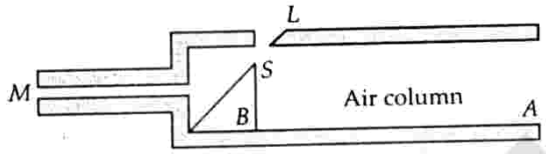


Fig. 15.19 Organ pipe.

When air is blown into the pipe through the *mouth piece* (*M*), it strikes against the slanting surface or *belevlled* (*B*). It is deflected upwards and issues out of the slit *S* in the form of a jet. This jet of air strikes against the sharp edge called the lip (*L*), setting it into vibrations. This produces a sound called *edge tone*, the frequency of which depends on the pressure of the air stream.

The sound waves travel down the pipe and get reflected at its closed or open end, producing longitudinal stationary waves. If the frequency of these waves is equal to the frequency of the edge tone, resonance occurs and a loud sound is produced.

If both the ends of the pipe are open, it is called an ***open pipe***. If one end of the pipe is closed, it is called ***closed pipe***.

30. *Describe the various modes of vibrations of an open organ pipe.*

Normal modes of vibration of an open organ pipe. Both the ends of an open organ pipe are open. The waves are reflected from these ends without change of type. However, the particles continue to move in the same direction even after the reflection of the waves. Consequently, the particles have the maximum displacements at the open ends. Hence antinodes are formed at the open ends. The various modes of vibration of an open organ pipe are shown in Fig. 15.20.

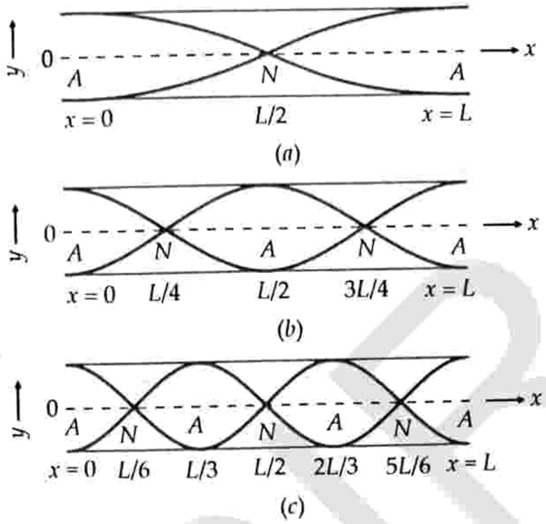


Fig. 15.20 Normal modes of vibration of an open pipe.

(*i*) ***First mode of vibration.*** In the simplest mode of vibration, there is one node in the middle and two antinodes at the ends of the pipe.

Here length of the pipe,

$$L = 2 \cdot \frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$

$$\therefore \quad \lambda_1 = 2L$$

Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu \quad \text{(say)}$$

This frequency is called *fundamental frequency* or *first harmonic.*

(*ii*) ***Second mode of vibration.*** Here antinodes at the open ends are separated by two nodes and one antinode.

$$L = 4 \cdot \frac{\lambda_2}{4} = \lambda_2$$

$$\text{Frequency,} \quad \nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}} = 2\nu$$

This frequency is called *first overtone* or *second harmonic.*

(*iii*) ***Third mode of vibration.*** Here the antinodes at the open ends are separated by three nodes and two antinodes.

$$L = 6 \cdot \frac{\lambda_3}{4} \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

$$\therefore \quad \text{Frequency,} \quad \nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called *second overtone* or *third harmonic.*

Hence various frequencies of an open organ pipe are in the ratio 1 : 2 : 3 : 4 :...... These are called harmonics.

31. *Prove analytically that in the case of an open organ pipe of length L, the frequencies of vibrating air column are given by $\nu = n\,(v / 2L)$, where n is an integer.*

Analytical treatment of stationary waves in an open organ pipe. Consider a cylindrical pipe of length L lying along the x-axis, with its open ends at $x = 0$ and $x = L$. The sound wave travelling along the pipe may be represented as

$$y_1 = A \sin(\omega t - kx)$$

The wave reflected from right open end may be represented as

$$y_2 = A \sin(\omega t + kx)$$

There is no phase reversal on reflection from the open end because it is a free or loose boundary. So the sign of A in the reflected wave is same as that in the incident wave.

By the principle of superposition, the resultant stationary wave is given by

$$y = y_1 + y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx)]$$
$$= 2A \sin \omega t \cos kx = (2A \cos kx) \sin \omega t$$

For all values of t, the resultant displacement is maximum (+ve or –ve) or antinodes are formed at the open ends *i.e.*, at $x = 0$ and $x = L$. This condition is satisfied if

$$\cos kL = \pm 1 \quad \text{or} \quad kL = n\pi,$$

where $n = 1, 2, 3, \ldots\ldots$

or $$\frac{2\pi}{\lambda} L = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

The frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}}$$

For $n = 1$, $$\nu_1 = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu \quad \text{(say)}$$

This is the smallest frequency of the stationary waves produced in the open pipe. It is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $$\nu_2 = \frac{2v}{2L} = 2\nu$$

(First overtone or second harmonic)

For $n = 3$, $$\nu_3 = \frac{3v}{2L} = 3\nu$$

(Second overtone or third harmonic)

and so on. The various modes of vibration of an open pipe are shown in Fig. 15.20.

32. *Give qualitative discussion of the different modes of vibration of a closed organ pipe.*

Normal modes of a closed organ pipe. In a closed organ pipe, one end of the pipe is open and the other end is closed. As the wave is reflected from the closed end, the direction of motion of the particles changes. The displacement is zero at the closed end *i.e.*, a node is formed at the closed end. The displacement of the particles is maximum at the open end, so an antinode is formed at the open end. The different modes of vibration of closed pipe are shown in Fig. 15.21.

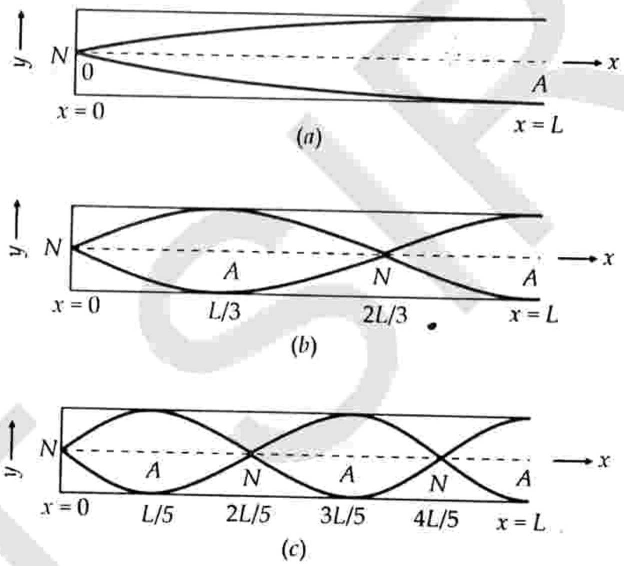


Fig. 15.21 Normal modes of vibration of closed pipe.

(*i*) ***First mode of vibration.*** In this simplest mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then

$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

This frequency is called *first harmonic* or *fundamental frequency*.

(*ii*) ***Second mode of vibration.*** In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end.

$$\therefore \quad L = \frac{3\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{4L}{3}$$

Frequency,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called *first overtone* or *third harmonic*.

(*iii*) ***Third mode of vibration.*** In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end.

$$\therefore \qquad L = \frac{5\lambda_3}{4} \qquad \text{or} \qquad \lambda_3 = \frac{4L}{5}$$

Frequency,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

Hence different frequencies produced in a closed organ pipe are in the ratio 1 : 3 : 5 : 7 : ...... *i.e.*, only odd harmonics are present in a closed organ pipe.

33. *Prove analytically that in the case of a closed organ pipe of length L, the frequencies of the vibrating air column are given by $v = (2n + 1)\ (v / 4L)$, where n is an integer.*

Analytical treatment of stationary waves in a closed organ pipe. Consider a cylindrical pipe of length L lying along the x-axis, with its closed end at $x = 0$ and open end at $x = L$. The sound wave sent along the pipe may be represented as

$$y_1 = A \sin(\omega t + kx)$$

The wave reflected from the closed end may be represented as

$$y_2 = -A \sin(\omega t - kx)$$

The negative sign before A is due to reversal of phase at the closed end.

By the principle of superposition, the resultant stationary wave is given by

$$y = y_1 + y_2 = A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$
$$= 2A \cos \omega t \sin kx = (2A \sin kx) \cos \omega t.$$

Clearly, $y = 0$ at $x = 0$ *i.e.*, a node is formed at the closed. The resultant displacement at $x = L$ will be maximum (+ve or –ve) because the open end is a free or loose boundary. This condition is satisfied if

$$\sin kL = \pm 1$$

or $$kL = (2n-1)\frac{\pi}{2} \qquad \text{where } n = 1, 2, 3, ...$$

or $$\frac{2\pi}{\lambda} L = (2n-1)\frac{\pi}{2} \qquad \text{or} \qquad \lambda_n = \frac{4L}{2n-1}$$

The corresponding frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{(2n-1)v}{4L} = \frac{(2n-1)}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

For $n = 1$,

$$\nu_1 = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

This is the smallest frequency of the stationary waves produced in the closed pipe. It is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $\quad \nu_2 = \frac{3v}{4L} = 3\nu$

(First overtone or third harmonic)

For $n = 3$, $\quad \nu_3 = \frac{5v}{4L} = 5\nu$

(Second overtone or fifth harmonic)

and so on. The various modes of vibration of a closed pipe are shown in Fig. 15.21.

Examples based on Organ Pipes and Rods Clamped in the Middle

FORMULAE USED

1. *In an organ pipe closed at one end*, only odd harmonics are present.

   Fundamental mode, $\nu_1 = \frac{v}{4L} = \nu$ (First harmonic)

   Second mode, $\nu_2 = 3\nu$

   (Third harmonic or first overtone)

   Third mode, $\nu_3 = 5\nu$

   (Fifth harmonic or second overtone)

   nth mode, $\nu_n = (2n - 1)\nu$

   [$(2n - 1)$th harmonic or $(n - 1)$th overtone]

2. *In an organ pipe open at both ends.* Both odd and even harmonics are present.

   Fundamental mode, $\nu_1' = \frac{v}{2L} = \nu'$ (First harmonic)

   Second mode, $\nu_2' = 2\nu'$

   (Second harmonic or first overtone)

   Third mode $\nu_3' = 3\nu'$

   (Third harmonic or second overtone)

   nth mode $\nu_n' = n\nu$

   [nth harmonic or $(n - 1)$th overtone]

   Clearly, $\nu_1' = 2\nu_1$

3. *Resonance tube.* If L_1 and L_2 are the first and second resonance lengths with a tuning fork of frequency ν, then the speed of sound,

   $v = 4\nu(L_1 + 0.3\,D)$,

   D = internal diameter of resonance tube

   or $\quad v = 2\nu(L_2 - L_1)$

   End correction $= 0.3\,D = \frac{L_2 - 3L_1}{2}$

UNITS USED

Velocity of sound v is in ms^{-1}, length of organ pipe L is in metre and frequency ν in Hz.

EXAMPLE 46. *What should be minimum length of an open organ pipe for producing a note of 110 Hz ? The speed of sound is 330 ms^{-1}.*

Solution. Frequency, $\nu = 110$ Hz,

Speed of sound, $v = 330\ ms^{-1}$

Fundamental frequency of an open organ pipe,

$$\nu = \frac{v}{2L}$$

$$\therefore \quad L = \frac{v}{2\nu} = \frac{330}{2 \times 110} = \mathbf{1.5\ m.}$$

EXAMPLE 47. *The length of an organ pipe open at both ends is 0.5 m. Calculate the fundamental frequency of the pipe, if the velocity of sound in air be 350 ms^{-1}. If one end of the pipe is closed, then what will be the fundamental frequency ?*

Solution. Speed of sound, $v = 350\ ms^{-1}$,

Length of the pipe, $L = 0.5$ m

Fundamental frequency of the open pipe,

$$\nu' = \frac{v}{2L} = \frac{350}{2 \times 0.5} = \mathbf{350\ Hz.}$$

Fundamental frequency of the closed pipe,

$$\nu = \frac{v}{4L} = \frac{350}{4 \times 0.5} = \mathbf{175\ Hz.}$$

EXAMPLE 48. *A pipe 30.0 cm long is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1.1 kHz source ? Will resonance with the same source be observed if one end of the pipe is closed ? Take the speed of sound in air as 330 ms^{-1}.* **[NCERT]**

Solution. Length of the pipe,

$$L = 30\ cm = 0.30\ cm$$

Speed of sound, $v = 330\ ms^{-1}$

Fundamental frequency of the open pipe,

$$\nu_1 = \frac{v}{2L} = \frac{330}{2 \times 0.30} = 550\ Hz$$

Second harmonic,

$$\nu_2 = 2\nu_1 = 2 \times 550 = 1100\ Hz$$

Third harmonic,

$$\nu_3 = 3\nu_1 = 3 \times 550 = 1650\ Hz$$

Fourth harmonic,

$$\nu_4 = 4\nu_1 = 4 \times 550 = 2200\ Hz, \text{ and so on.}$$

Clearly, a source of frequency 1.1 kHz (or 1100 Hz) will resonantly excite the *second harmonic* of the open pipe.

If one end of the pipe is closed, its fundamental frequency becomes

$$\nu_1' = \frac{v}{4L} = \frac{330}{4 \times 0.30} = 275\ Hz$$

As only odd harmonics are present in a closed pipe, so

Third harmonic,

$$\nu_3' = 3\nu_1' = 3 \times 275 = 825\ Hz$$

Fifth harmonic,

$$\nu_5' = 5\nu_1' = 5 \times 275 = 1375\ Hz, \text{ and so on.}$$

As no frequency of the closed pipe matches with the source frequency of 1.1 kHz, so no resonance will be observed with the source, the moment one end of the pipe is closed.

EXAMPLE 49. *Find the ratio of the length of a closed pipe to that of an open pipe in order that the second overtone of the former is in unison with the fourth overtone of the latter.*

Solution. Fundamental frequency of a closed organ pipe, $\nu = \frac{v}{4L}$

Fundamental frequency of an open organ pipe,

$$\nu' = \frac{v}{2L'}$$

Second overtone of the closed pipe $= 5\nu = \frac{5v}{4L}$

Fourth overtone of the open pipe $= 5\nu' = \frac{5v}{2L'}$

As the two overtones are in unison, therefore

$$\frac{5v}{4L} = \frac{5v}{2L'} \quad \text{or} \quad \frac{L}{L'} = \frac{2}{4} = \frac{1}{2}$$

or $\quad L : L' = \mathbf{1 : 2.}$

EXAMPLE 50. *An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of the open pipe ?* **[IIT 96]**

Solution. Fundamental frequency of open pipe,

$$\nu_o = \frac{v}{2L}$$

Frequency of third harmonic of closed pipe,

$$\nu_c = \frac{3v}{4L}$$

$$\therefore \quad \frac{\nu_c}{\nu_o} = \frac{3}{2} \quad \text{or} \quad \nu_c = \frac{3}{2}\nu_o$$

Given $\quad \nu_c = \nu_o + 100$

$$\therefore \quad \frac{3}{2}\nu_o = \nu_o + 100 \quad \text{or} \quad \nu_o = \mathbf{200\ Hz.}$$

EXAMPLE 51. *The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound in air = 330 ms^{-1}.* **[IIT 97]**

Solution. Let lengths of the open and closed organ pipes be L_0 and L_c respectively.

Fundamental frequency of closed pipe,

$$\nu_c = \frac{v}{4L_c}$$

$$\therefore \quad L_c = \frac{v}{4\nu_c}$$

But $\quad v = 330 \text{ ms}^{-1}, \quad \nu_c = 110 \text{ Hz}$

$$\therefore \quad L_c = \frac{330}{4 \times 110} = 0.75 \text{ m} = \mathbf{75 \text{ cm.}}$$

Frequency of first overtone of open organ pipe,

$$\nu_0 = 2 \times \frac{v}{2L_0} = \frac{v}{L_0}$$

Frequency of first overtone of closed pipe,

$$\nu_c = 3 \times \frac{v}{4L_c} = 3 \times 110 = 330 \text{ Hz.}$$

But $\nu_0 - \nu_c = 2.2$ Hz

$$\therefore \quad \frac{v}{L_0} - 330 = 2.2 \qquad \text{or} \qquad \frac{v}{L_0} = 332.2$$

or $$L_0 = \frac{v}{332.2} = \frac{330}{332.2} = 0.99 \text{ m} = \mathbf{99 \text{ cm.}}$$

Example 52. *A well with vertical sides and water at the bottom resonates at 7 Hz and at no other lower frequency. The air in the well has density 1.10 kg m^{-3} and bulk modulus of 1.33×10^5 Nm^{-2}. How deep is the well ?*

Solution. For air $\rho = 1.10 \text{ kg m}^{-3}$,

$$\kappa = 1.33 \times 10^5 \text{ Nm}^{-2}$$

$\therefore$ Speed of sound in air,

$$v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{1.33 \times 10^5}{1.10}} = 347.7 \text{ ms}^{-1}$$

A well can be regarded as a closed organ pipe. So its fundamental frequency,

$$\nu = \frac{v}{4L}$$

or $$L = \frac{v}{4\nu} = \frac{347.7}{4 \times 7} = \mathbf{12.41 \text{ ms}^{-1}}.$$

Example 53. *A resonance air column resonates with a tuning fork of 512 Hz at length 17.4 cm. Neglecting the end correction, deduce the speed of sound in air.*

Solution. Here $\nu = 512$ Hz, $L_1 = 17.4$ cm

When end correction is neglected, speed of sound in air is given by

$$v = 4\nu L_1 = 4 \times 512 \times 17.4 = 35635.2 \text{ cm s}^{-1}$$
$$= \mathbf{356.35 \text{ ms}^{-1}}.$$

Example 54. *A resonance tube is resonated with tuning fork of frequency 512 Hz. Two successive lengths of the resonated air-column are 16.0 cm and 51.0 cm. The experiment is performed at the room temperature of 40°C. Calculate the speed of sound at 0°C and the end correction.*

Solution. Here $\nu = 512$ Hz, $L_1 = 16.0$ cm $= 0.16$ m,

$$L_2 = 51.0 \text{ cm} = 0.51 \text{ m}$$

Speed of sound at 40°C,

$$v = 2\nu(L_2 - L_1) = 2 \times 512 \times (0.51 - 0.16)$$
$$= 358.4 \text{ ms}^{-1}$$

Speed of sound at 0°C is given by

$$v_0 = v - 0.61\, t = 358.4 - 0.61 \times 40$$
$$= 358.4 - 24.4 = \mathbf{334 \text{ ms}^{-1}}.$$

End correction

$$= \frac{L_2 - 3L_1}{2} = \frac{51.0 - 48.0}{2} = \mathbf{1.5 \text{ cm.}}$$

Example 55. *Determine the possible harmonics in the longitudinal vibration of a rod clamped in the middle.*

Solution. Consider a rod of length L clamped in the middle. As shown in Fig. 15.22(*a*), it has one node in the middle and two antinodes at its free ends in the fundamental mode.

$$\therefore \quad L = 2 \cdot \frac{\lambda_1}{4} \qquad \text{or} \qquad \lambda_1 = 2L$$

Fundamental frequency or first harmonic,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \nu \text{ (say)}$$

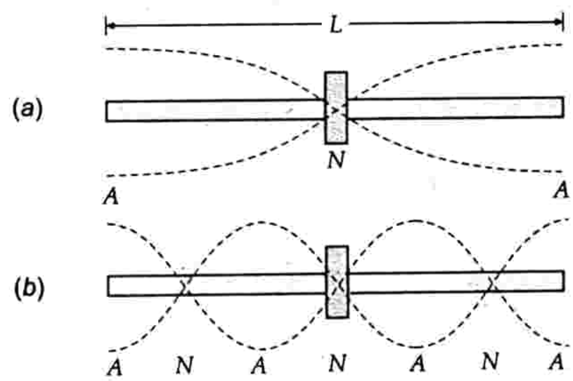


Fig. 15.22

In the second mode, there is an additional node and antinode on the two sides of the clamp, as shown in Fig. 15.22(*b*).

$$\therefore \quad L = 6 \cdot \frac{\lambda_2}{4} \qquad \text{or} \qquad \lambda_2 = \frac{2L}{3}$$

and $$\nu_2 = \frac{v}{\lambda_2} = \frac{3v}{2L} = 3\nu$$

This is called *third harmonic* or *first overtone.*

Similarly, for third mode,

$$\nu_3 = \frac{5v}{2L} = 5\nu$$

This is called *fifth harmonic* or *second overtone*.

Hence $\nu_1 : \nu_2 : \nu_3 : ... = 1 : 3 : 5 : , ...$

EXAMPLE 56. *A brass rod 1 metre long is firmly clamped in the middle and one end is stroked by a resined cloth. What is the pitch of the note you will hear ? Young's modulus for brass = 10^{12} dyn cm^{-2} and density = 9 g cm^{-3}.*

Solution. Here $Y = 10^{12}$ dyn cm^{-2}, $\rho = 9$ g cm^{-3}

Speed of sound in the brass rod,

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{12}}{9}} = \frac{10^6}{3} \text{ cm s}^{-1}.$$

Length of rod,

$$L = 1 \text{ m} = 200 \text{ cm}$$

Fundamental note,

$$\nu = \frac{v}{\lambda} = \frac{v}{2L} = \frac{10^6}{3 \times 200} = \mathbf{1666.67\ Hz.}$$

PROBLEMS FOR PRACTICE

1. An open organ pipe produces a note of frequency 512 Hz at 15°C, calculate the length of the pipe. Velocity of sound at 0°C is 335 ms^{-1}. (**Ans.** 0.336 m)
2. Find the frequencies of the fundamental note and the first overtone in an open air column and a closed air column of length 34 cm. The velocity of sound at room temperature is 340 ms^{-1}. [**Ans.** (*i*) 500 Hz, 1000 Hz (*ii*) 250 Hz, 750 Hz]
3. Prove that a pipe of length 2 L open at both ends has same fundamental frequency as another pipe of length L closed at the other end. Also, state whether the total sound will be identical for two pipes. (**Ans.** No)
4. The fundamental frequency of a closed organ pipe is equal to the first overtone of an open organ pipe. If the length of the open pipe is 60 cm, what is the length of the closed pipe ? (**Ans.** 15 cm)
5. The fundamental tone produced by an organ pipe has a frequency of 110 Hz. Some other frequencies in the notes produced by this pipe are 220, 440, 550, 660 Hz. Is this pipe open at both ends or open at one end and closed at the other ? Calculate the effective length of the pipe. Speed of sound = 330 ms^{-1}. (**Ans.** Pipe is open at both ends, 1.5 m)
6. The fundamental frequency of an open organ pipe is 300 Hz. The frequency of the first overtone of another closed organ pipe is the same as the frequency of the first overtone of open pipe. What are the lengths of the pipes ? The speed of sound = 330 ms^{-1}. (**Ans.** 55.0 cm, 41.25 cm)
7. Find the ratio of length of a closed organ pipe to that of open pipe in order that the second overtone of the former is in unison with fourth overtone of the latter. (**Ans.** 3 : 2)
8. A tuning fork of frequency 341 Hz is vibrated just over a tube of length 1 m. Water is being poured gradually in the tube. What height of water column will be required for resonance ? The speed of sound in air is 341 ms^{-1}. (**Ans.** 25 cm or 75 cm)
9. A resonance air column shows resonance with a tuning fork of frequency 256 Hz at column lengths 33.4 cm and 101.8 cm. Find (*i*) end-correction and (*ii*) the speed of sound in air. [**Ans.** (*i*) 0.8 cm (*ii*) 350.2 ms^{-1}]
10. A metallic bar clamped at its middle point vibrates with a frequency ν when it is rubbed at one end. If its length is doubled, what will be its natural frequency of vibration ? (**Ans.** ν / 2)

HINTS

1. Velocity of sound at 15°C will be

$$v = v_0 + 0.61t = 335 + 0.61 \times 15 = 344.15 \text{ ms}^{-1}$$

Fundamental frequency of an open organ pipe,

$$\nu = \frac{v}{4L} \quad \therefore \quad L = \frac{v}{4\nu} = \frac{344.15}{4 \times 512} = \mathbf{0.336\ m.}$$

3. Fundamental frequency of closed pipe of length L,

$$\nu = \frac{v}{4L}$$

Fundamental frequency of an open pipe of length $2L$,

$$\nu' = \frac{v}{2 \times 2L} = \frac{v}{4L}. \text{ Clearly, } \nu = \nu'$$

In an open organ pipe all harmonics are present whereas in closed organ pipe, only odd harmonics are present. Hence the quality of sound will be different for the two pipes.

4. Fundamental frequency of a closed organ pipe,

$$\nu_1 = \frac{v}{4L}$$

Fundamental frequency of an open organ pipe,

$$\nu_1' = \frac{v}{2L'}$$

Frequency of first overtone of open organ pipe,

$$\nu_2' = 2\nu_1' = \frac{v}{L'}$$

Given $\nu_1 = \nu_2'$

$$\therefore \quad \frac{v}{4L} = \frac{v}{L'} \quad \text{or} \quad L = \frac{L'}{4} = \frac{60}{4} = \mathbf{15\ cm.}$$

9. End correction $= \frac{L_2 - 3L_1}{2} = \frac{101.8 - 3 \times 33.4}{2}$

$= \frac{1.6}{2} = \mathbf{0.8\ cm.}$

Speed of sound,

$$v = 2\nu (L_2 - L_1) = 2 \times 256 \times (1.018 - 0.334)$$
$$= 2 \times 256 \times 0.684 = \mathbf{350.2\ ms^{-1}}.$$

10. Natural frequency, $\nu = \frac{v}{\lambda} = \frac{v}{2L}$

On doubling the length, frequency is halved.

15.22 BEATS

34. *What are beats ? What is the essential condition for the formation of beats ?*

Beats. When two sound waves of slightly different frequencies travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls (technically known as waxing and waning of sound) alternately with time. *These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequencies are called beats.* One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.

Beat frequency = Difference in frequencies of the two superposing waves

$$\nu_{beat} = \nu_1 - \nu_2$$

Essential condition for the formation of beats. For beats to be audible, the difference in the frequency of the two sound waves should not exceed 10. If the difference is more than 10, we shall hear more than 10 beats per second. But due to persistence of hearing, our ear is not able to distinguish between two sounds as separate if the time interval between them is less than (1/10)th of a second. Hence beats heard will not be distinct if the number of beats produced per second is more than 10.

35. *Explain the formation of beats by graphical method.*

Formation of beats by graphical method. In Fig. 15.23(*a*), the full line curve is the displacement-time curve of a wave of frequency ν_1 and the dashed curve is for a wave of frequency ν_2. Here ν_1 is slightly greater than ν_2, so the first wave is slightly smaller than second.

At time t_1, the two waves meet in the same phase at a given point. They reinforce to produce maximum sound intensity. With the passage of time, the phase difference between the two waves increases and so the two curves gradually get more and more out of step.

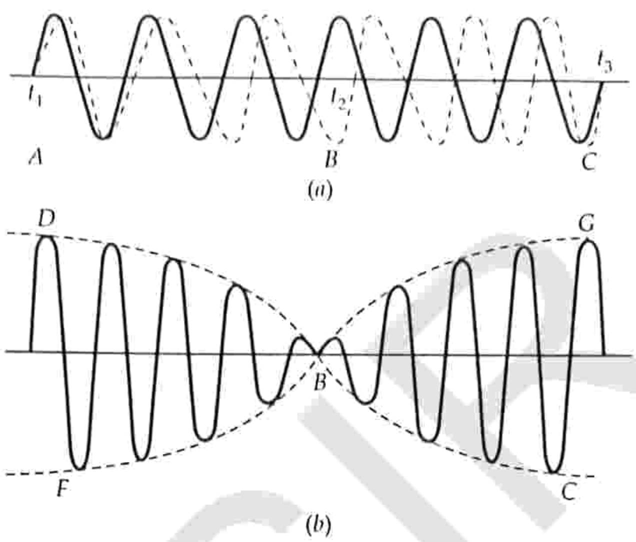


Fig. 15.23 Formation of beats graphically.

At time t_2, the two waves are in exactly opposite phases. This happens when one wave gains half a vibration over the other. Now they produce minimum sound intensity. Now the phase difference goes on decreasing with time. At time t_3 one wave gains one full vibration on the other and the two waves are again in same phase and produce maximum intensity, and so on. The resultant wave, obtained by the algebraic sum of the displacements of the two waves, is shown by full line curve in Fig. 15.23(*b*). The dashed envelopes above and below it show how the amplitude of the resultant wave varies with time. The time interval from t_1 to t_3 is *one beat period,* because during this interval only one beat is formed. Moreover, during this time interval, the first wave completes $(\nu + 1)$ oscillations while the second wave completes ν oscillations. Thus beat frequency is equal to the difference in frequencies of the two superposing waves.

36. *Explain the formation of beats analytically. Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves.*

Analytical treatment of beats. Consider two harmonic waves of frequencies ν_1 and ν_2 (ν_1 being slightly greater than ν_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented as

$$y_1 = A \sin \omega_1 t = A \sin 2\pi \nu_1 t$$
$$y_2 = A \sin \omega_2 t = A \sin 2\pi \nu_2 t$$

By the principle of superposition, the resultant displacement at the given point will be

$$y = y_1 + y_2 = A \sin 2\pi \nu_1 t + A \sin 2\pi \nu_2 t$$
$$= 2A \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2}\right) t \,.\, \sin 2\pi \left(\frac{\nu_1 + \nu_2}{2}\right) t$$

If we write

$$\nu_{mod} = \frac{\nu_1 - \nu_2}{2} \quad \text{and} \quad \nu_{av} = \frac{\nu_1 + \nu_2}{2}$$

then $\quad y = 2A\cos(2\pi\nu_{mod}\, t)\sin(2\pi\nu_{av}\, t)$

or $\quad y = R\sin(2\pi\nu_{av}\, t)$

where $R = 2A\cos(2\pi\nu_{mod}\, t)$ is the amplitude of the resultant wave. As ν_1 is slightly greater than ν_2, so $\nu_{mod} << \nu_{av}$ *i.e.*, R varies very slowly with time. Hence the above equation represents a wave of periodic rapid oscillation of average frequency ν_{av} 'modulated' by a slowly varying oscillation of frequency ν_{mod}.

The amplitude R of the resultant wave will be maximum, when

$$\cos 2\pi\nu_{mod}\, t = \pm 1$$

or $\quad 2\pi\nu_{mod}\, t = n\pi$

or $\quad \pi(\nu_1 - \nu_2)\, t = n\pi$

or $\quad t = \frac{n}{\nu_1 - \nu_2} = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}, \ldots$

∴ Time interval between two successive maxima

$$= \frac{1}{\nu_1 - \nu_2}$$

Similarly, the amplitude R will be minimum, when

$$\cos 2\pi\nu_{mod}\, t = 0$$

or $\quad 2\pi\nu_{mod}\, t = (2n+1)\pi/2$

or $\quad \pi(\nu_1 - \nu_2)t = (2n+1)\pi/2$

or $\quad t = \frac{(2n+1)}{2(\nu_1 - \nu_2)} = \frac{1}{\nu_1 - \nu_2}, \frac{3}{2(\nu_1 - \nu_2)}, \frac{5}{2(\nu_1 - \nu_2)}, \ldots$

∴ The time interval between successive minima

$$= \frac{1}{\nu_1 - \nu_2}$$

Clearly, both maxima and minima of intensity occur alternately. Technically, one maximum of intensity followed by a minimum is called a *beat*. Hence the time interval between two successive beats or the *beat period* is

$$t_{beat} = \frac{1}{\nu_1 - \nu_2}$$

The *number of beats produced per second is called* ***beat frequency***.

$$\nu_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \nu_{beat} = \nu_1 - \nu_2$$

∴ Beat frequency = Difference between the frequencies of two superposing waves.

37. *How will you experimentally demonstrate the phenomenon of beats in sound ?*

Experimental demonstration of beats in sound. As shown in Fig. 15.24, mount two tuning forks A and B of exactly the same frequency on two sound boxes, placed with their open ends facing each other.

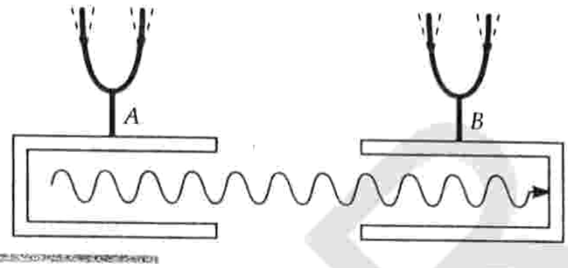


Fig. 15.24 Experimental demonstrations of beats.

Now stick a little wax to the prong of one of them to as to slightly reduce its frequency. Set the two tuning forks into vibrations. The two tuning forks will produce sound waves of different frequencies. The intensity of the resulting sound will increase and decreasing periodically with time. We will actually hear beats. By counting the number of beats heard in a given interval of time, we can calculate the beat frequency and hence can determine the difference $(\nu_1 - \nu_2)$ between the frequencies of the two tuning forks.

15.23 PRACTICAL APPLICATIONS OF BEATS

38. *Explain some practical applications of beats.*

Practical applications of beats. (*i*) **Determination of an unknown frequency.** Suppose ν_1 is the known frequency of tuning fork A and ν_2 is the unknown frequency of tuning fork B. When the two tuning forks are sounded together, suppose they produce b beats per second. Then

$$\nu_2 = \nu_1 + b \quad \text{or} \quad \nu_1 - b$$

The exact frequency may be determined by any of the following two methods :

(*a*) ***Loading method.*** Attach a little wax to the prong of the tuning fork B. Again, find the number of beats produced per second. If the frequency of B is greater than that of A *i.e.*, $(\nu_1 + b)$, then the attaching of a little wax lowers its frequency and reduces the difference in frequencies of A and B. This would decrease the beat frequency.

Hence, if *the beat frequency decreases on loading the prong of the tuning fork of the unknown frequency, then the unknown frequency is greater than the known frequency.* That is,

$$\nu_2 = \nu_1 + b$$

On the other hand, if the frequency of B is less than that of A *i.e.*, $(\nu_1 - b)$, then the attaching of a little wax further lowers its frequency and increases the difference in frequencies of A and B. This would increase the beat frequency.

Hence, *if the beat frequency increases on loading the prong of the tuning fork of the unknown frequency, then the unknown frequency is less than the known frequency.* That is, $\nu_2 = \nu_1 - b$

(*b*) ***Filing method.*** If a prong of the tuning fork *B* is filed, its frequency increases. Again, note the number of beats produced per second.

If on filing the prong of *B*, the beat frequency decreases, then $\nu_2 = \nu_1 - b$

If on filing the prong of *B*, the beat frequency increases, then $\nu_2 = \nu_1 + b$

(*ii*) **For tuning musical instruments.** Musicians use the beat phenomenon in tuning their musical instruments. If an instrument is sounded against a standard frequency and tuned until the beats disappear, then the instrument is in tune with the standard frequency.

(*iii*) **For producing colourful effects in music.** Sometimes, a rapid succession of beats is knowingly introduced in music. This produces an effect similar to that of human voice and is appreciated by the audience.

(*iv*) **For detection of marsh gas in mines.** Here we use two small organ pipes tuned to the same frequency. One pipe contains pure dry air and the other ordinary mine air.

If any marsh gas or methane appears in the mine, the density of the mine air in the second pipe decreases which slightly changes its frequency of vibration. When sounded with the first organ pipe, it gives rise to beats. The miners are thus warned well in advance of the explosive marsh gas.

(*v*) **Use in electronics.** It is difficult to make low frequency oscillators. In practice, two high frequency oscillators with a small difference in their frequencies are used. Their low beat frequency $(\nu_1 - \nu_2)$ serves the purpose of a low frequency oscillator.

Examples based on Beats Formation

FORMULAE USED

1. Beat frequency = Number of beats per second
   = Difference in frequencies of two sources
   or $b = (\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$
2. $\nu_2 = \nu_1 \pm b$
3. If the prong of tuning fork is *filed*, its frequency increases. If the prong of a tuning fork is *loaded* with a little wax, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

UNITS USED

All frequencies are in Hz or s^{-1}.

EXAMPLE 57. *The points of the prongs of a tuning fork B originally in unison with a tuning fork A of frequency 384 are filed and the fork produces 3 beats per second, when sounded together with A. What is the pitch of B after filing ?*

Solution. Frequency of tuning fork $A = 384$ Hz.

As tuning fork *B* is in unison with *A*, so its original frequency = 384 Hz

Beat frequency $= 3\ s^{-1}$

Possible frequencies of *B* after filing

$$= 384 \pm 3 = 387 \text{ or } 381 \text{ Hz}$$

As the frequency increases on filing, so frequency of *B* after filing = **387 Hz**.

EXAMPLE 58. *A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. What is the frequency of the unknown fork ?* **[AIEEE 02]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 288 \pm 4 = 292$ or 284 Hz

On loading with wax, the frequency decreases, the beat frequency also decreases to 2.

∴ Unknown frequency = **292 cps** (higher one).

EXAMPLE 59. *A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.* **[IIT]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 310 \pm 4 = 314$ or 306 Hz

Out of these two possible frequencies, one must be the initial value and the other final value. As frequency increases on filing, therefore

initial unknown frequency = **306 Hz.**

EXAMPLE 60. *A fork of unknown frequency when sounded with one of frequency 288 Hz gives 4 beats per second and when loaded with a piece of wax again gives 4 beats per second. How do you account for this and what was the unknown frequency ?* **[IIT]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 288 \pm 4 = 292$ or 284 Hz.

As the beat frequency remains unchanged even on loading the tuning fork, of the two possible frequencies one must be initial frequency and the other the final one. But the frequency of a tuning fork decreases on loading, therefore

initial unknown frequency = **292 Hz.**

EXAMPLE 61. Two tuning forks A and B produce 4 beats/second. On loading B with wax, 6 beats/second were heard. If the quantity of wax is reduced, the number of beats per second again becomes 4. Find the frequency of B if the frequency of A is 256 Hz.

Solution. Frequency of tuning fork $A = 256$ Hz

Beat frequency $= 4\ s^{-1}$

$\therefore$ Possible frequencies of B

$$= 256 \pm 4 = 260 \text{ or } 252 \text{ Hz.}$$

As some wax continues to remain attached finally with the tuning fork B and beats/second $= 4$, so the final frequency must be less than the initial frequency.

$\therefore$ Initial frequency of $B =$ **260 Hz.**

EXAMPLE 62. A tuning fork produces 4 beats/s when sounded with a tuning fork of frequency 512 Hz. The same tuning fork when sounded with another tuning fork of frequency 514 Hz produces 6 beats/s. Find the frequency of the tuning fork.

Solution. Let the frequency of the tuning fork $= \nu$

It produces 4 beats/s with a tuning fork of frequency 512 Hz

$\therefore \quad \nu = 512 \pm 4 = 516 \quad \text{or} \quad 508 \text{ Hz} \quad ...(1)$

It also produces 6 beats/s with a tuning fork of frequency 514 Hz

$\therefore \quad \nu = 514 \pm 6 = 520 \quad \text{or} \quad 508 \text{ Hz} \quad ...(2)$

Equations (1) and (2) show that the common frequency is 508.

$\therefore \quad \nu =$ **508 Hz.**

EXAMPLE 63. A tuning fork of known frequency of 256 Hz makes 5 beat s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. What was the frequency of the piano string before increasing the tension ? **[AIEEE 03]**

Solution. Frequency of a piano string

$$= 256 \pm 5 = 261 \quad \text{or} \quad 251 \text{ Hz}$$

When the tension in the piano string is increased, its frequency increases ($\nu \propto \sqrt{T}$). If the original frequency is 251 Hz (the lower one), the beat frequency should decrease on increasing the tension. This is given to be the case as the beat frequency decreases from 5 beats s^{-1} to 2 beats s^{-1}.

$\therefore$ Original frequency of piano = **251 Hz**.

EXAMPLE 64. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2 ? **[AIEEE 05]**

Solution. Frequency of fork 2

$= $ Frequency of fork 1 $\pm$ Beat frequency

$= 200 \pm 4 = 204 \quad \text{or} \quad 196$ Hz

When some tape is attached to the prong of fork 2, its frequency decreases, but the beat frequency increases from 4 to 6.

$\therefore$ Frequency of fork 2 = **196 Hz** (the lower one).

EXAMPLE 65. A set of 24 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats per second with the preceding one and the last sounds the octave of the first, find the frequencies of the first and the last forks.

Solution. Let frequency of first fork $= \nu$

Then frequency of second fork $= \nu + 4$

Frequency of third fork $= \nu + 2 \times 4$

Frequency of fourth fork $= \nu + 3 \times 4$

Frequency of 24th fork $= \nu + 23 \times 4$

But the frequency of the last is the octave of the first.

$\therefore \quad 2\nu = \nu + 23 \times 4 \quad \text{or} \quad \nu = 92 \text{ Hz}$

Frequency of the first fork

$= \nu =$ **92 Hz.**

Frequency of the last fork

$= 2\nu =$ **184 Hz.**

EXAMPLE 66. In an experiment, it was found that a tuning fork and a sonometer wire gave 5 beats per second both when the length of the wire was 1 m and 1.05 m. Calculate the frequency of the fork.

Solution. Here $L_1 = 1$ m, $L_2 = 1.05$ m,

Beat frequency $= 5\ s^{-1}$

As $\quad \nu \propto \dfrac{1}{L} \quad \therefore \quad \dfrac{\nu_1}{\nu_2} = \dfrac{L_2}{L_1}$

But $\quad L_1 < L_2 \quad \therefore \quad \nu_1 > \nu_2$

If ν is the frequency of the tuning fork, then

$$\nu_1 = \nu + 5 \quad \text{and} \quad \nu_2 = \nu - 5$$

Hence $\quad \dfrac{\nu + 5}{\nu - 5} = \dfrac{1.05}{1}$

or $\quad \nu + 5 = 1.05\,\nu - 5.25 \quad \text{or} \quad 0.05\,\nu = 10.25$

or $\quad \nu = \dfrac{10.25}{0.05} =$ **205 Hz.**

EXAMPLE 67. A tuning fork of frequency 200 Hz is in unison with a sonometer wire. How many beats per second will be heard if the tension of the wire were increased by 2% ?

Solution. Here $\nu_1 = 200$ Hz

As the tension in the wire is increased by 2%, therefore if $T_1 = 100$ units, then $T_2 = 102$ units

Now $\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{102}{100}} = \left(1 + \frac{2}{100}\right)^{1/2}$

$$= 1 + \frac{1}{2} \times \frac{2}{100} = 1.01$$

$\therefore \quad \nu_2 = 1.01\,\nu_1 = 1.01 \times 200 = 202 \text{ Hz}$

Beat frequency

$$= \nu_2 - \nu_1 = 202 - 200 = \mathbf{2\ Hz.}$$

EXAMPLE 68. *The two parts of sonometer wire divided by a movable knife differ by 2 mm and produce one beat per second when sounded together. Find their frequencies if the whole length of the wire is one metre.*

Solution. Let L_1 and L_2 be the lengths of the two parts of a sonometer wire divided by a movable knife edge. Then

$$L_1 + L_2 = 100 \text{ cm}$$

and $\quad L_1 - L_2 = 2 \text{ mm} = 0.2 \text{ cm}$

Adding and subtracting, we get

$$L_1 = 50.1 \text{ cm} \quad \text{and} \quad L_2 = 49.9 \text{ cm}$$

Let ν_1 and ν_2 be the frequencies of the sonometer wire corresponding to lengths L_1 and L_2 respectively. Then the number of beats produced per second,

$$\nu_2 - \nu_1 = 1 \qquad ...(1)$$

According to the law of length of a stretched string,

$$\frac{\nu_2}{\nu_1} = \frac{L_1}{L_2} = \frac{50.1}{49.9}$$

or $\quad \nu_2 = \frac{50.1}{49.9}\nu_1 \qquad ...(2)$

From (1) and (2), we get

$$\frac{50.1}{49.9}\nu_1 - \nu_1 = 1$$

or $\quad \frac{50.1\,\nu_1 - 49.9\,\nu_1}{49.9} = 1$

or $\quad \frac{0.2\,\nu_1}{49.1} = 1$

$\therefore \quad \nu_1 = \frac{49.9}{0.2} = \mathbf{249.5\ Hz.}$

From (1), $\quad \nu_2 = \nu_1 + 1 = 249.5 + 1 = \mathbf{250.5\ Hz.}$

EXAMPLE 69. *Two similar sonometer wires of the same material produce 2 beats per second. The length of one is 50 cm and that of the other is 50.1 cm. Calculate the frequencies of the two wires.*

Solution. Frequency, $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{k}{l}$,

where $k = \frac{1}{2}\sqrt{\frac{T}{m}}$ = a constant

Now $\quad \nu_1 = \frac{k}{L_1} \quad \text{and} \quad \nu_2 = \frac{k}{L_2}$

$\therefore \quad \nu_1 - \nu_2 = k\left(\frac{1}{L_1} - \frac{1}{L_2}\right)$

But $L_1 = 50$ cm, $\quad L_2 = 50.1$ cm, $\quad \nu_1 - \nu_2 = 2\ \text{s}^{-1}$

$\therefore \quad 2 = k\left(\frac{1}{50} - \frac{1}{50.1}\right) \quad \text{or} \quad k = 50100$

Hence $\quad \nu_1 = \frac{k}{L_1} = \frac{50100}{50} = \mathbf{1002\ Hz}$

and $\quad \nu_2 = \frac{k}{L_2} = \frac{50100}{50.1} = \mathbf{1000\ Hz.}$

EXAMPLE 70. *A tuning fork of unknown frequency vibrates in unison with a wire of certain length stretched under a tension of 5 kg f. It produces 6 beats per second with the same wire, when tension is changed to 4.5 kg f. Find the frequency of tuning fork.*

Solution. Fundamental frequency of a stretched string,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = k\sqrt{T},$$

where $k = \frac{1}{2L\sqrt{m}}$ = a constant

When $T = 5$ kg f $= 5 \times 9.8$ N, the wire is in unison with a tuning fork of frequency ν. Therefore,

$$\nu = k\sqrt{5 \times 9.8}$$

When the tension decreases, frequency of vibration decreases. When the tension reduces to 4.5 kg f or 4.5×9.8 N, it produces 6 beats per second. Clearly, the frequency of vibration is

$$\nu' = \nu - 6$$

Also $\quad \nu' = k\sqrt{4.5 \times 9.8}$

$\therefore \quad k\sqrt{4.5 \times 9.8} = k\sqrt{5 \times 9.8} - 6$

or $\quad 6.64\,k = 7k - 6$

or $\quad k = \frac{6}{0.36} = 16.67$

$\therefore \quad \nu = k\sqrt{5 \times 9.8}$

$$= 16.67 \times 7 = \mathbf{116.72\ Hz.}$$

EXAMPLE 71. *Calculate the speed of sound in a gas in which two sound waves of wavelengths 1.00 m and 1.01 m produce 24 beats in 6 seconds.*

Solution. Let v be the sound in the gas. Then

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{1.00} \quad \text{and} \quad \nu_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

Beat frequency,

$$\nu_1 - \nu_2 = 24/6 = 4\ \text{s}^{-1}$$

$$\therefore \quad v\left[\frac{1}{1.00}-\frac{1}{1.01}\right]=4 \quad \text{or} \quad \frac{v \times 0.01}{1.01}=4$$

or $$v=\frac{4 \times 1.01}{0.01}=\mathbf{404\ ms^{-1}}.$$

EXAMPLE 72. Two air columns (of resonance tubes) 100 cm and 101 cm long give 17 beats in 20 seconds, when each is sounding its fundamental mode. Calculate the velocity of sound.

Solution. Here $L_1 = 100$ cm $= 1.00$ m,

$$L_2 = 101 \text{ cm} = 1.01 \text{ m}$$

Beat frequency $= \nu_1 - \nu_2$

$$\frac{17}{20}=\frac{v}{4L_1}-\frac{v}{4L_2}=\frac{v}{4}\left[\frac{1}{1.00}-\frac{1}{1.01}\right]=\frac{v \times 0.01}{4 \times 1.01}$$

or $$v=\frac{17 \times 4 \times 1.01}{20 \times 0.01}=\mathbf{343.4\ ms^{-1}}.$$

EXAMPLE 73. Two tuning forks A and B give 5 beats per second. A resounds with a closed column of air 15 cm long and B with an open column 30.5 cm long. Calculate their frequencies. Neglect end correction.

Solution. Let ν_1 and ν_2 be the frequencies of the tuning forks A and B respectively.

As fork A resounds with a closed air column of length 15 cm or 0.15 m, so

$$\nu_1=\frac{v}{4L_1}=\frac{v}{4 \times 0.15}=\frac{v}{0.60}$$

Again, fork B resounds with an open air column of length 30.5 cm or 0.305 m, so

$$\nu_2=\frac{v}{2L_2}=\frac{v}{2 \times 0.305}=\frac{v}{0.61}$$

Beat frequency $= \nu_1 - \nu_2$

$$5=\frac{v}{0.60}-\frac{v}{0.61}=\frac{v \times 0.01}{0.60 \times 0.61}$$

or $$v=\frac{5 \times 0.60 \times 0.61}{0.01}=183 \text{ ms}^{-1}$$

$$\therefore \quad \nu_1=\frac{183}{0.60}=\mathbf{305\ Hz}$$

and $$\nu_2=\frac{183}{0.61}=\mathbf{300\ Hz.}$$

EXAMPLE 74. At 16°C, two open end organ pipes, when sounded together produce 34 beats in 2 seconds. How many beats per second will be produced, if the temperature rises to 51°C ? Neglect the increase in length of the pipes.

Solution. Number of beats produced per second at 16°C $= 34/2 = 17$

Let L_1, L_2 = lengths of the two organ pipes,

ν_1, ν_2 = lowest frequencies emitted by the pipes at 16°C,

ν'_1, ν'_2 = lowest frequencies emitted by the pipes at 51°C,

v_{16}, v_{51} = velocities of sound at 16°C and 51°C respectively.

$$\therefore \quad \nu_1=\frac{v_{16}}{2L_1} \quad \text{and} \quad \nu_2=\frac{v_{16}}{2L_2}$$

Beat frequency at 16°C,

$$\nu_1 - \nu_2 = 17$$

or $$\frac{v_{16}}{2L_1}-\frac{v_{16}}{2L_2}=17$$

$$v_{16}\left[\frac{1}{2L_1}-\frac{1}{2L_2}\right]=17 \qquad \text{...(1)}$$

Let the number of beats per second at 51°C $= b$

Then $$\nu'_1 - \nu'_2 = b$$

or $$\frac{v_{51}}{2L_1}-\frac{v_{51}}{2L_2}=b$$

$$v_{51}\left[\frac{1}{2L_1}-\frac{1}{2L_2}\right]=b \qquad \text{...(2)}$$

Dividing equation (2) by (1), we get

$$\frac{v_{51}}{v_{16}}=\frac{b}{17}$$

or $$\sqrt{\frac{51+273}{16+273}}=\frac{b}{17} \qquad [\because v_t \propto \sqrt{t+273}]$$

or $$b=17 \times \sqrt{\frac{324}{289}}$$

$$=17 \times \frac{18}{17}=\mathbf{18\ s^{-1}}.$$

EXAMPLE 75. A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15°C. When the temperature falls to 10°C, the two produce 3 beats per second. Find the frequency of the fork.

Solution. Let frequency of the tuning fork $= \nu$

Frequency of air column at 15°C,

$$\nu_{15} = \nu + 4$$

Frequency of air column at 10°C,

$$\nu_{10} = \nu + 3$$

As $$v = \nu\lambda$$

$$\therefore \quad v_{15} = (\nu+4)\lambda \quad \text{and} \quad v_{10} = (\nu+3)\lambda$$

Hence $\frac{v_{15}}{v_{10}} = \frac{\nu + 4}{\nu + 3}$ ...(1)

Also $\frac{v_{15}}{v_{10}} = \sqrt{\frac{273 + 15}{273 + 10}} = \sqrt{\frac{288}{283}}$ ...(2)

From (1) and (2), we have

$$\frac{\nu + 4}{\nu + 3} = \left(\frac{288}{283}\right)^{1/2} = \left(1 + \frac{5}{283}\right)^{1/2} \simeq 1 + \frac{1}{2} \times \frac{5}{283}$$

or $\frac{(\nu + 3) + 1}{\nu + 3} = 1 + \frac{5}{566}$ or $1 + \frac{1}{(\nu + 3)} = 1 + \frac{5}{566}$

$\therefore$ $\frac{1}{(\nu + 3)} = \frac{5}{566}$ or $(\nu + 3) = \frac{566}{5} = 113.2$

$\therefore$ $\nu = 110.2 \text{ Hz} \simeq$ **110 Hz.**

Problems For Practice

1. When two tuning forks were sounded together, 20 beats were produced in 10 seconds. On loading one of the forks with wax, the number of beats increases. If the frequency of unloaded fork is 512 Hz, calculate the frequency of other. (**Ans.** 510 Hz)
2. A tunning fork of unknown frequency gives 4 beats per second when sounded with a fork of frequency 320 Hz. When loaded with little wax, it gives 3 beats per second. Find the unknown frequency. **[Central Schools 12]** (**Ans.** 324 Hz)
3. A tuning fork *A* makes 4 beats per second with a fork *B* of frequency 256 Hz. *A* is filed and the beats occur at shorter interval, find its original frequency. **[MNREC 85]** (**Ans.** 260 Hz)
4. A set of 25 tuning forks is arranged in order of decreasing frequency. Each fork gives 3 beats with succeeding one. The first fork is octave of the last. Calculate the frequency of the first and 16th fork. (**Ans.** 144 Hz, 99 Hz)
5. The string of a violin emits a note of 440 Hz at its correct tension. The string is bit taut and produces 4 beats per second with a tuning fork of frequency 440 Hz. Find the frequency of the note emitted by this taut string. (**Ans.** 444 Hz)
6. A tuning fork when vibrating along with a sonometer produces 6 beats per second when the length of the wire is either 20 cm or 21 cm. Find the frequency of the tuning fork. (**Ans.** 246 Hz)
7. A 70 cm long sonometer wire is in unison with a tuning fork. If the length of the wire is decreased by 1.0 cm, it produces 4 beats per second with the same tuning fork. Find the frequency of the tuning fork. (**Ans.** 276 Hz)
8. When two tuning forks are sounded together, 4 beats per second are heard. One of the forks is in unison with 0.96 m length of a sonometer wire and the other is in unison with 0.97 m length of the same wire. Calculate the frequency of each. (**Ans.** 388 Hz, 384 Hz)
9. Two perfectly identical wires are in unison. If the tension in one wire is increased by 1%, then on sounding them together, 3 beats are produced in 2 seconds. Calculate the frequency of each wire. (**Ans.** 300 Hz)
10. *A* and *B* are two wires whose fundamental frequencies are 256 and 382 Hz respectively. How many beats in two seconds will be heard by the third harmonic of *A* and second harmonic of *B* ? (**Ans.** 8)
11. In an experiment, it was found that a tuning fork and a sonometer wire gave 4 beats per second, both when the length of the wire was 1 m and 1.05 m. Calculate the frequency of the fork. (**Ans.** 164 Hz)
12. A tuning fork of frequency 300 Hz resonates with an air-column closed at one end at 27°C. How many beats will be heard in the vibrations of the fork and the air-column at 0°C ? End-correction is negligible. (**Ans.** 14 s^{-1})

Hints

1. Beat frequency $= \frac{20}{10} = 2\text{ s}^{-1}$

   Possible unknown frequencies $= 512 \pm 2$

   $= 514$ or 510 Hz

   As loading decreases the frequency, also the beat frequency increases, so unknown frequency = 510 Hz (lower one).

4. Let the frequency of last fork $= \nu$

   Then frequency of first fork $= 2\nu$

   Frequency of second fork $= 2\nu - 3$

   Frequency of third fork $= 2\nu - 2 \times 3$

   Frequency of 25th fork $= 2\nu - 24 \times 3 = \nu$

   $\therefore$ $\nu = 72$

   $\therefore$ Frequency of first fork

   $= 2\nu =$ **144 Hz**

   Frequency of 16th fork

   $= 144 - 15 \times 3 =$ **99 Hz.**

5. When the string is taut, its frequency increases because $\nu \propto \sqrt{T}$.

   As the taut string produces 4 beats per second with a tuning fork of frequency 440, so its frequency = 440 + 4 = **444 Hz.**

6. Here $\nu + 6 = \frac{1}{2 \times 20}\sqrt{\frac{T}{m}}$

and $\nu - 6 = \frac{1}{2 \times 21}\sqrt{\frac{T}{m}}$

$\therefore \quad \frac{\nu + 6}{\nu - 6} = \frac{21}{20}$ or $(\nu + 6) \times 20 = (\nu - 6)\, 21$

On solving, ν = **246 Hz.**

7. Let ν be the frequency of the tuning fork. Then

$$\nu = \frac{1}{2 \times 0.70}\sqrt{\frac{T}{m}}$$

When the length decreases to 69 cm or 0.69 m, frequency increases.

$\therefore \quad \nu + 4 = \frac{1}{2 \times 0.69}\sqrt{\frac{T}{m}}$

Hence $\frac{\nu}{\nu + 4} = \frac{0.69}{0.70}$

or $\quad 0.70\,\nu = 0.69\,(\nu + 4)$

or $\quad \nu = \frac{4 \times 0.69}{0.01}$ = **276 Hz.**

8. Frequency of first fork,

$$\nu_1 = \frac{1}{2 \times 0.96}\sqrt{\frac{T}{m}}$$

Frequency of second fork,

$$\nu_2 = \frac{1}{2 \times 0.97}\sqrt{\frac{T}{m}}$$

$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{97}{96}$ or $96\nu_1 = 97\nu_2$

Clearly, $\nu_1 > \nu_2 \quad \therefore \quad \nu_1 - \nu_2 = 4$ or $\nu_1 = \nu_2 + 4$

Hence $96(\nu_2 + 4) = 97\nu_2$

or $\quad \nu_2$ = **384 Hz.**

$\nu_1 = \nu_2 + 4$ = **388 Hz.**

9. Here $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$

and $\nu + \frac{3}{2} = \frac{1}{2L}\sqrt{\frac{1.01\,T}{m}}$

$\therefore \quad \frac{\nu + 1.5}{\nu} = \sqrt{1.01} = (1 + 0.01)^{1/2}$

$= 1 + \frac{1}{2} \times 0.01 = 1.005$

On solving ν = **300 Hz.**

10. Beat frequency

$= 3\nu_1 - 2\nu_2 = 3 \times 256 - 2 \times 382$

$= 768 - 764 = 4\ s^{-1}$

Number of beats produced in 2 seconds

$= 4 \times 2 =$ **8.**

11. As $\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1}$

$\therefore \quad \frac{\nu + 4}{\nu - 4} = \frac{1.05}{1} = \frac{21}{20}$

or $\quad 20\nu + 80 = 21\nu - 84$

or $\quad \nu$ = **164 Hz.**

12. Frequency of air column at 27°C,

$$\nu = \frac{v_{27}}{4L} = 300$$

Now $\frac{v_0}{v_{27}} = \sqrt{\frac{0 + 273}{27 + 273}} = \left(1 - \frac{27}{300}\right)^{1/2} = 0.954$

$\therefore \quad v_0 = 0.954 \times v_{27}$

Frequency of air column at 27°C,

$$\nu' = \frac{v}{4L} = \frac{0.954 \times v_{27}}{4L} = 0.954 \times 300 = 286$$

$\therefore$ Beats produced per second = 300 − 286 = **14.**

15.24 DOPPLER EFFECT

39. *What is Doppler effect ? Give an example.*

Doppler effect. Whenever there is a relative motion between the source of sound, the observer and the medium ; the frequency of sound as received by the observer is different from the frequency of sound emitted by the source. *The apparent change in the frequency of sound when the source, the observer and the medium are in relative motion is called Doppler effect.* For example, consider a man standing on a railway platform. When a train, blowing its whistle, approaches him, the pitch of the whistle appears to rise and it suddenly appears to drop as the engine moves away from him. Similar effect is observed when the source is at rest and the observer moves towards or away from the source.

Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves such as microwaves, radiowaves and visible light. However, Doppler effect is noticeable only when the relative velocity between the source and the observer is an appreciable fraction of the wave velocity.

For the waves which require a medium for their propagation, the apparent frequency depends on *three* factors : (*i*) velocity of the source, (*ii*) velocity of the observer and (*iii*) velocity of medium or wind.

40. *Derive an expression for the apparent frequency of sound as heard by a stationary observer in a still medium, when the source is moving towards the observer with a uniform velocity. Hence write the expression for the apparent frequency when the source moves away from the stationary observer.*

Apparent frequency when the source moves towards the stationary observer. Consider a source S moving with speed v_s towards an observer O who is at rest with respect to the medium, as shown in Fig. 15.25. If ν is the frequency of vibration of the source, then it sends out sound (or compressional) waves with speed v at a regular interval of $T = 1/\nu$.

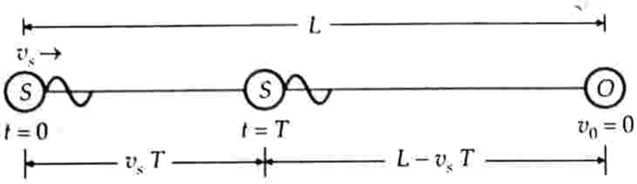


Fig. 15.25 A source S moving with speed v_s towards a stationary observer O.

At time $t = 0$, suppose the source is at distance L from the observer and emits a compression pulse. It reaches the observer at time,

$$t_1 = \frac{L}{v}$$

The source emits next compression pulse after a time T. In the mean time, the source has moved a distance $v_s T$ towards the observer and is now at distance $L - v_s T$ from the observer. The second compression pulse reaches the observer at time,

$$t_2 = T + \frac{L - v_s T}{v}$$

The time interval between two successive compression pulses or the period of the wave as detected by the observer is

$$T' = t_2 - t_1 = T + \frac{L - v_s T}{v} - \frac{L}{v}$$

$$= \left(1 - \frac{v_s}{v}\right) T = \frac{v - v_s}{v} T$$

The apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v}{v - v_s} \cdot \frac{1}{T}$$

or
$$\nu' = \frac{v}{v - v_s} \nu \qquad \text{...(1)}$$

Clearly, $\nu' > \nu$. Hence *the pitch of sound appears to increase when the source moves towards the stationary observer.*

Apparent frequency when the source moves away from the stationary observer. If the source moves away from the observer with speed v_s, then the apparent frequency of sound can be obtained by replacing v_s by $-v_s$ in equation (1).

Thus
$$\nu' = \frac{v}{v + v_s} \nu \qquad \text{...(2)}$$

Clearly, $\nu' < \nu$. Hence *the pitch of sound appears to decrease when the source of sound moves away from the stationary observer.*

41. *Derive an expression for the apparent frequency of the sound, when the observer moves towards a stationary source of sound. Hence write the expression for the apparent frequency when the observer moves away from the stationary source.*

Apparent frequency when the observer moves towards the stationary source. As shown in Fig. 15.26, now we consider the case when the source S remains stationary with respect to the medium and the observer O moves towards the source with speed v_0. If ν is the frequency of vibration of the source, then it sends out compression pulses at a regular interval of $T = 1/\nu$, which travel through the medium with speed v.

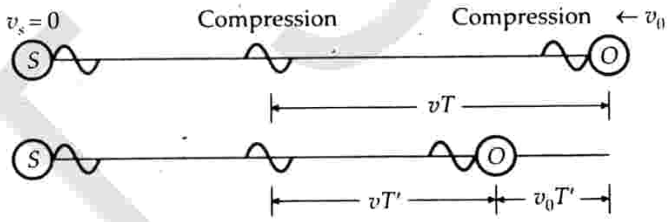


Fig. 15.26 An observer moving with speed v_0 towards the stationary source S.

At any instant, the separation between two compression pulses is $\lambda = vT$. So when the observer receives a compression pulse, the next compression pulse is a distance vT away from him. This second compression pulse moves towards the observer with speed v and also the observer moves towards it with speed v_0. Hence the observer will receive the second compression wave a time T' after receiving the first pulse, where

$$T' = \frac{vT}{v + v_0}$$

This is the period of the wave as detected by the observer. Hence the apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v + v_0}{v} \frac{1}{T}$$

or
$$\nu' = \frac{v + v_0}{v} \nu \qquad \text{...(3)}$$

Clearly, $\nu' > \nu$. Thus *the pitch of sound appears to increase when the observer moves towards the source.*

Apparent frequency when the observer moves away from the stationary source. If the observer moves away from the stationary source with speed v_0, then the apparent frequency of sound can be obtained by replacing v_0 by $-v_0$ in equation (3). Thus

$$\nu' = \frac{v - v_0}{v}\,\nu \qquad \text{...(4)}$$

Clearly $\nu' < \nu$. Thus *the pitch of sound appears to decrease when the observer moves away from the stationary source.*

42. *Obtain an expression for the observed frequency of the sound produced by a source when both observer and source are in motion and the medium at rest.*

Apparent frequency when both the source and observer are in motion. As shown in Fig. 15.27, consider the case when both the source and the observer are moving towards each other with speeds v_s and v_0 respectively. If ν is the frequency of the source, it sends out compression pulses through the medium at regular intervals of $T = 1/\nu$.

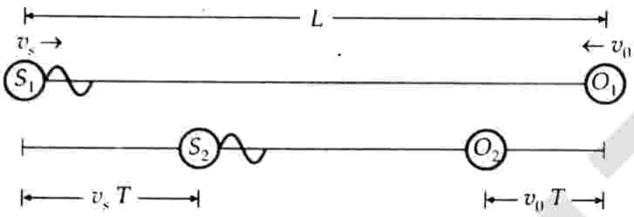


Fig. 15.27 A source and observer both moving towards each other.

At time $t = 0$, the observer is at O_1 and the source at S_1 and the distance between them is L when the source emits the first compression pulse. Since the observer is also moving towards the source, so the speed of the wave relative to the observer is $(v + v_0)$. Therefore, the observer will receive the first compression pulse at time,

$$t_1 = \frac{L}{v + v_0}$$

At time $t = T$, both the source and observer have moved towards each other covering distances $S_1S_2 = v_s T$ and $O_1O_2 = v_0 T$ respectively. The new distance between the source and the observer is

$$S_2O_2 = L - (v_s + v_0)T$$

The second compression pulse will reach the observer at time,

$$t_2 = T + \frac{L - (v_s + v_0)T}{v + v_0}$$

The time interval between two successive compression pulses or the period of the wave as recorded by the observer is

$$T' = t_2 - t_1$$

$$= T + \frac{L - (v_s + v_0)T}{v + v_0} - \frac{L}{v + v_0}$$

$$= \left(1 - \frac{v_s + v_0}{v + v_0}\right)T = \left(\frac{v - v_s}{v + v_0}\right)T$$

Hence the apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v + v_0}{v - v_s} \cdot \frac{1}{T}$$

or
$$\nu' = \frac{v + v_0}{v - v_s}\,\nu \qquad \text{...(5)}$$

When the source moves towards the observer and the observer moves away from the source. In this case, the apparent frequency can be obtained by replacing v_0 by $-v_0$ in equation (5). Thus

$$\nu' = \frac{v - v_0}{v - v_s}\,\nu \qquad \text{...(6)}$$

If the medium also moves with a velocity v_m in the direction of propagation of sound, then

$$\nu' = \frac{v + v_m - v_0}{v + v_m - v_s}\,\nu$$

If the medium moves with a velocity v_m in the opposite direction of sound, then

$$\nu' = \frac{v - v_m - v_0}{v - v_m - v_s}\,\nu.$$

43. *Doppler effect in sound is asymmetric. What do you mean by this statement ?*

Doppler effect in sound is asymmetric. Suppose a source of sound moves towards a stationary observer with the speed v'. Then the observed frequency will be

$$\nu' = \frac{v}{v - v'}\,\nu$$

Now if the observer moves towards the stationary source with the same speed v', then the observed frequency will be

$$\nu'' = \frac{v + v'}{v}\,\nu$$

Clearly, $\nu' \neq \nu''$. Thus the observed frequency is not same when the observer is stationary and the source moves towards it or when the source is stationary and the observer moves towards it with the same speed. For this reason, the *Doppler effect in sound is said to asymmetric. However, the Doppler effect in light is symmetric.* This is because sound or mechanical waves, in general, have a velocity relative to the medium through which they travel whereas light or electromagnetic waves travel quite independent of it.

For Your Knowledge

- The observed frequency depends on the actual velocities of the source and the observer and not on their relative velocities.
- The motion of the source brings about a change in the wavelength of the sound waves and hence there is a change in the observed frequency.
- The motion of the observer merely changes the rate at which the sound waves are received by him. The observer intercepts more waves (when he approaches) or fewer waves (when he recedes) each second. The wavelength of the sound waves remains unaffected.
- The apparent frequency is larger than the actual frequency, if the separation between the source and the observer is decreasing and is smaller if the separation is increasing.
- **No Doppler effect is observed** *i.e.*, there is no shift in frequency in the following situations : (*i*) When both the source and the observer move in the same direction with the same speed. (*ii*) When either the source or the observer is at the centre of a circle and the other is moving along it with an uniform speed. (*iii*) When both the source and the observer are at rest and the wind alone is blowing.
- The Doppler formula for apparent frequency is applicable when $v_0 < v$ and $v_s < v$. It does not hold when the speed of the source or the observer becomes equal to greater than the speed of the wave.
- When the observer is at rest and the source moves with a supersonic speed (a speed greater than the speed of sound), the resultant wave motion is a conical wave called a **shock wave**. It is due to the shock wave that we hear a sudden and violent sound, called **sonic boom**, when a supersonic jet plane passes by.
- The ratio of the speed of the source to the speed of sound (v/v_s) is called **Mach number**. Shock waves are produced by objects moving with Mach number greater than one.

Examples based on Doppler Effect in Sound

FORMULAE USED

1. If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency $\nu' = \dfrac{v + v_m - v_0}{v + v_m - v_s} \times \nu$
2. If the medium is at rest $(v_m = 0)$, then $\nu' = \dfrac{v - v_0}{v - v_s} \times \nu$
3. All the velocities are taken *positive* in the *source to observer* ($S \rightarrow O$) direction and *negative* in the opposite ($O \rightarrow S$) direction.

UNITS USED

Velocities v, v_0, v_s and v_m are in ms^{-1} and frequencies ν and ν' in Hz.

EXAMPLE 76. A source and an observer are approaching one another with the relative velocity 40 ms^{-1}. If the true source frequency is 1200 Hz, deduce the observed frequency under the following conditions :

(i) All velocity is in the source alone.

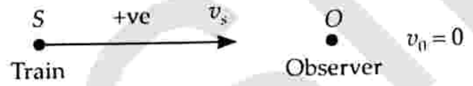
(ii) All velocity is in the observer alone.

(iii) The source moves in air at 100 ms^{-1} towards the observer, but the observer also moves with the velocity v_0 in the same direction.

Solution. Here $\nu = 1200$ Hz, $v = 340\ ms^{-1}$,

Relative velocity $= 40\ ms^{-1}$

(*i*) Here source moves towards the stationary observer, $v_s = +40\ ms^{-1}$, $v_0 = 0$

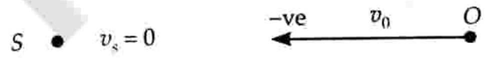


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 40} \times 1200$$

$$= \frac{340 \times 1200}{300} = \mathbf{1360\ Hz.}$$

(*ii*) Here observer moves towards the stationary source,

$$v_0 = -40\ ms^{-1},\ v_s = 0.$$

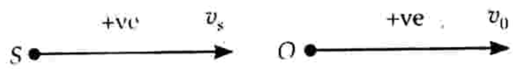


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 40}{340 - 0} \times 1200$$

$$= \frac{380}{340} \times 1200 = \mathbf{1341\ Hz.}$$

(*iii*) Here observer and source move in the same direction,

$$v_s = 100\ ms^{-1},\ v_0 = 100 - 40 = 60\ ms^{-1}$$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 60}{340 - 100} \times 1200$$

$$= \frac{280}{240} \times 1200 = \mathbf{1400\ Hz.}$$

EXAMPLE 77. A railway engine and a car are moving on parallel tracks in opposite directions with speed of 144 kmh^{-1} and 72 kmh^{-1}, respectively. The engine is continuously sounding a whistle of frequency 500 Hz. The velocity of sound is 340 ms^{-1}. Calculate the frequency of sound heard in the car when

(i) the car and the engine are approaching each other,

(ii) the two are moving away from each other.

Solution. Here $v_s = 144 \text{ km h}^{-1}$

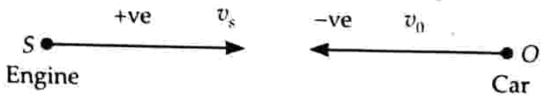
$$= \frac{144 \times 1000}{3600} = 40 \text{ ms}^{-1}$$

and $$v_0 = 72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} = 20 \text{ ms}^{-1}$$

$$\nu = 500 \text{ Hz}, \quad v = 340 \text{ ms}^{-1}$$

(*i*) When the car and the engine approach each other,

$$v_s = +40 \text{ ms}^{-1}, \quad v_0 = -20 \text{ ms}^{-1}$$

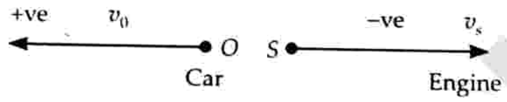


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 20}{340 - 40} \times 500$$

$$= \frac{360}{300} \times 500 = \textbf{600 Hz.}$$

(*ii*) When the car and the engine are moving away from each other,

$$v_s = -40 \text{ ms}^{-1}, \quad v_0 = +20 \text{ ms}^{-1}$$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 20}{340 + 40} \times 500$$

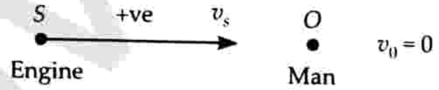
$$= \frac{320}{380} \times 500 = \textbf{421 Hz.}$$

EXAMPLE 78. *The sirens of two fire engines have a frequency of 600 Hz each. A man hears the sirens from the two engines, one approaching him with a speed of 36 km h^{-1} and the other going away from him at a speed of 54 km h^{-1}. What is the difference in frequency of two sirens heard by the man ? Take the speed of sound to be 340 ms^{-1}.* **[Punjab 91]**

Solution. Here $v = 340 \text{ ms}^{-1}$, $\nu = 600$ Hz

For the engine approaching the man :

$$v_s = +36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}, \quad v_0 = 0$$

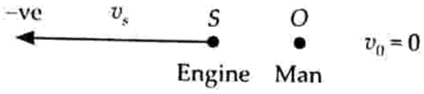


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 600$$

$$= \textbf{618.2 Hz}$$

For the engine going away from the man :

$$v_s = -54 \text{ km h}^{-1} = -15 \text{ ms}^{-1}, \quad v_0 = 0$$



$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 15} \times 600 = \textbf{576.6 Hz}$$

Difference in frequencies

$$= \nu' - \nu'' = 618.2 - 576.6 = \textbf{41.6 Hz.}$$

EXAMPLE 79. *An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ?* **[AIEEE 05]**

Solution. Here observer moves towards the stationary source.

$$\therefore \quad v_0 = -v/5, \quad v_s = 0$$

Apparent frequency

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{v + v/5}{v - 0} \times \nu$$

$$= \frac{6}{5}\nu = 1.2\,\nu$$

The percentage increase in apparent frequency,

$$\frac{\nu' - \nu}{\nu} \times 100 = \frac{1.2\,\nu - \nu}{\nu} \times 100 = \textbf{20\%.}$$

EXAMPLE 80. *An observer standing on a railway crossing receives frequencies of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. The speed of sound in air is 300 ms^{-1}.* **[IIT 05]**

Solution. When the train approaches the stationary observer, the apparent frequency is

$$\nu' = \frac{v}{v - v_s} \times \nu$$

or $$2.2 = \frac{300}{300 - v_s} \times \nu \quad ...(1)$$

When the train recedes from the stationary observer, the apparent frequency is

$$\nu'' = \frac{v}{v + v_s} \times \nu$$

or $$1.8 = \frac{300}{300 + v_s} \times \nu \quad ...(2)$$

Dividing (1) by (2), we get

$$\frac{2.2}{1.8} = \frac{300 + v_s}{300 - v_s} \quad \text{or} \quad \frac{11}{9} = \frac{300 + v_s}{300 - v_s}$$

or $3300 - 11\,v_s = 2700 + 9\,v_s$

or $20 v_s = 600$

or $v_s = \textbf{30 ms}^{-1}$.

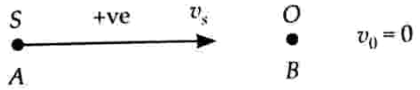
EXAMPLE 81. *On a quiet day, two persons A and B, each sounding a note of frequency 580 Hz, are standing a few metres apart. Calculate the number of beats heard by each in one second when A moves towards B with a velocity of 4 ms^{-1}. (Speed of sound in air = 330 ms^{-1}.)*

Solution. In one case, A can be regarded as a source of sound moving towards observer B.

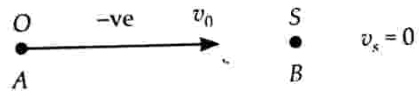
$$v_s = +4\ \text{ms}^{-1}, \quad v_0 = 0$$

$$\therefore \quad \nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{330 - 0}{330 - 4} \times 580$$

$$= \frac{330}{320} \times 580 = 587\ \text{Hz}$$



In another case, A can be regarded as observer moving towards stationary source B.



$$v_0 = -4\ \text{ms}^{-1}, \quad v_s = 0$$

$$\therefore \quad \nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{330 + 4}{330 - 0} \times 580$$

$$= \frac{334}{330} \times 580 = 587\ \text{Hz}$$

Number of beats heard per second by A

$$= \nu'' - \nu = 587 - 580 = 7.$$

Number of beats heard per second by B

$$= \nu' - \nu = 587 - 580 = 7.$$

EXAMPLE 82. *Find the velocity of source of sound, when the frequency appears to be (i) double (ii) half the original frequency to a stationary observer. Velocity of sound = 330 ms^{-1}.*

Solution. (*i*) Here $\nu' = 2\nu,\ v_0 = 0,\ v = 330\ \text{ms}^{-1}$

As $\quad \nu' = \dfrac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad 2\nu = \dfrac{v - 0}{v - v_s} \times \nu$

or $\quad v - v_s = v/2$

$$v_s = v/2 = 330/2 = \mathbf{165\ ms^{-1}}.$$

Positive value of v_s shows that the source is moving towards the observer.

(*ii*) Here $\nu' = \nu/2, \quad v_0 = 0, \quad v_s = 330\ \text{ms}^{-1}$

As $\quad \nu' = \dfrac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad \dfrac{\nu}{2} = \dfrac{v - 0}{v - v_s} \times \nu$

or $\quad v - v_s = 2v$

or $\quad v_s = -v = \mathbf{-330\ ms^{-1}}.$

Negative value of v_s shows that the source is moving away from the observer.

EXAMPLE 83. *A train stands at a platform blowing a whistle of frequency 400 Hz in still air.*

(*i*) *What is the frequency of the whistle heard by a man running (a) towards the engine at 10 ms^{-1}, (b) away from the engine at 10 ms^{-1} ?*

(*ii*) *What is speed of sound in each case ?*

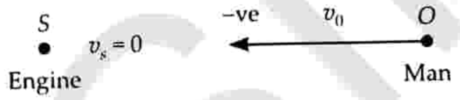
(*iii*) *What is the wavelength of sound received by the running man in each case ?*

Take speed of sound in still air = 340 ms^{-1}

Solution. Here $\nu = 400$ Hz, $v = 340\ \text{ms}^{-1}$

(*i*) (*a*) When the man runs towards the engine,

$$v_0 = -10\ \text{ms}^{-1}, \quad v_s = 0$$

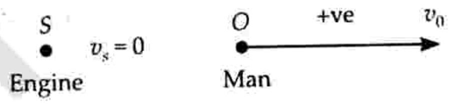


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 10}{340 - 0} \times 400$$

$$= \frac{350}{340} \times 400 = \mathbf{411.8\ Hz.}$$

(*b*) When the man runs away from the engine,

$$v_0 = +10\ \text{ms}^{-1}, \quad v_s = 0$$



$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 10}{340 - 0} \times 400$$

$$= \frac{330}{340} \times 400 = \mathbf{388.2\ Hz.}$$

(*ii*) (*a*) When the man runs towards the engine, relative velocity of sound,

$$v' = v + v_0$$

$$= 340 + 10 = \mathbf{350\ ms^{-1}}.$$

(*b*) When the man runs away from the engine, relative velocity of sound,

$$v' = v - v_0$$

$$= 340 - 10 = \mathbf{330\ ms^{-1}}.$$

(*iii*) The wavelength of sound is not affected by the motion of the listener. Its value is

$$\lambda = \frac{v}{\nu} = \frac{340}{400} = \mathbf{0.85\ m.}$$

EXAMPLE 84. *Consider a source moving towards an observer at the speed $v_s = 0.95\,v$. Deduce the observed frequency if the original frequency is 500 Hz. Think what would happen if $v_s > v$. (Jet planes moving faster than sound are so common). Here v is the velocity of sound.*

Solution. Here the source is moving towards the observer at rest.

$\therefore \quad v_s = +0.95\,v, \quad v_0 = 0, \quad \nu = 500$ Hz

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{v - 0}{v - 0.95\,v} \times 500 = \frac{500}{0.05}$$

$= \mathbf{10{,}000\ Hz.}$

If $v_s > v$, $v - v_s$ is negative and as such ν' is negative which has no physical meaning. Doppler formula is not applicable when the velocity of source exceeds the velocity of the wave.

EXAMPLE 85. *A machine gun is mounted on an armoured car. The gun can point (a) in the direction of motion or against the direction of motion of the car. The muzzle speed of the bullet equals the speed of sound in still air i.e., 340 ms^{-1}. If the car moves with a speed of 20 ms^{-1}, find out the sign and magnitude of the time difference between the bullet's arrival (t_b) and the arrival of the sound of firing (t_s) at a target 500 m away from the car at the instant of firing in each case.*

Solution. Speed of sound, $v = 340$ ms^{-1}

Muzzle speed of the bullet, $v_b = 340$ ms^{-1}

Speed of the car, $v_c = 20$ ms^{-1}

Distance of the target $= 500$ m

(*a*) *When the gun points in the direction of motion of the car.* Here the car is moving towards the target.

$\therefore$ Effective speed of the bullet

$= v_b + v_c = 360$ ms^{-1}

Time taken by the bullet to hit the target,

$$t_b = \frac{500}{360} = 1.3889 \text{ s}$$

Time taken by sound to reach the target,

$$t_s = \frac{500}{340} = 1.4706 \text{ s}$$

$\therefore \quad t_b - t_s = 1.3889 - 1.4706 = \mathbf{-0.0817\ s.}$

(*b*) *When the gun points against the direction of motion of the car.* Here the car is moving away from the target.

$\therefore$ Effective speed of the bullet

$= v_b - v_c = 340 - 20 = \mathbf{320\ ms^{-1}}$

Time taken by the bullet to hit the target,

$$t_b' = \frac{500}{320} = 1.5625 \text{ s}$$

$\therefore \quad t_b' - t_s = 1.5625 - 1.405 = \mathbf{0.0919\ s.}$

EXAMPLE 86. *An observer is moving towards a wall at 2 ms^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the wall. Calculate the beat frequency between these two sounds, if the true frequency of the source is 680 Hz. Velocity of sound = 340 ms^{-1}.*

Solution. When the observer receives sound directly from the source, he is moving away from the source, so $\quad v_0 = +2$ ms^{-1}, $\quad v_s = 0$

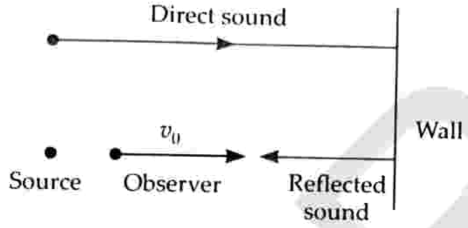


Fig. 15.28

Apparent frequency,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 2}{340 - 0} \times 680 = 676 \text{ Hz.}$$

When the observer receives reflected sound, he is moving towards the wall (source), so

$v_0 = -2$ ms^{-1}, $\quad v_s = 0$

Apparent frequency,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 2}{340 - 0} \times 680 = \mathbf{684\ Hz}$$

Beat frequency $= \nu'' - \nu' = 684 - 676 = \mathbf{8\ Hz.}$

EXAMPLE 87. *A rocket is moving at a speed of 200 ms^{-1} towards a stationary target. While moving, it emits a sound wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (a) the frequency of the sound wave as detected by a detector attached to the target and (b) the frequency of the echo as detected by a detector attached to the rocket.*

[NCERT ; Chandigarh 07]

Solution. (*a*) Here the observer is at rest and the source is moving with a speed of 200 ms^{-1}. As the speed of the source is comparable to that of the sound wave, so the observed frequency is

$$\nu' = \nu\left(1 - \frac{v_s}{v}\right)^{-1} = 1000\left(1 - \frac{200}{330}\right)^{-1}$$

$$= \frac{1000 \times 330}{130} = \mathbf{2538.5\ Hz.}$$

(*b*) Now the target is the source (as it is the source of echo) and the rocket's detector is the observer who intercepts the echo of frequency ν'. Hence the frequency of the echo as detected by a detector attached to the rocket is

$$\nu'' = \frac{v + v_0}{v - v_s} \times \nu = \frac{330 + 200}{330 - 0} \times 2538.5$$

$\simeq \mathbf{4077\ Hz.}$

EXAMPLE 88. *A siren is fitted on a car going towards a vertical wall at a speed of 36 kmh^{-1}. A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave*

(a) coming directly from the siren to the person, and

(b) coming after reflection. Take the speed of sound to be 340 ms^{-1}. Actual frequency of siren = 500 Hz.

Solution. The situation is shown in Fig. 15.29.

Here $v_s = 36$ kmh^{-1} = 10 ms^{-1}, $v = 340$ ms^{-1}, $\nu = 500$ Hz

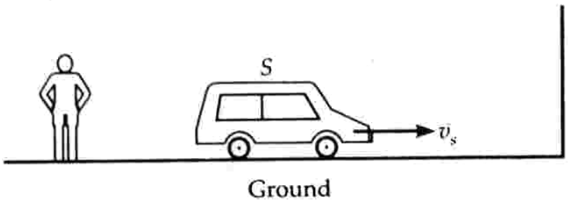


Fig. 15.29

(*a*) For the sound coming directly from the siren to the observer, source (siren) is moving away from the observer, so

$$v_s = -10 \text{ ms}^{-1}, \quad v_0 = 0$$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 10} \times 500 = 485.7 \text{ Hz}$$

(*b*) For the sound received by the wall, source (siren) is moving towards the wall (observer), so

$$v_s = +10 \text{ ms}^{-1}, \; v_0 = 0$$

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 500 = 515.2 \text{ Hz}$$

As the wall reflects the sound without changing the frequency, so apparent frequency of the wave after reflection = **515.2 Hz.**

EXAMPLE 89. *If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 330 ms^{-1}.*

Solution. Apparent frequency heard by the person is given by

$$\nu' = \left(\frac{v - v_0}{v}\right)\nu \quad \text{or} \quad \frac{\nu'}{\nu} = \frac{v - v_0}{v}$$

But $\quad \frac{\nu'}{\nu} = \frac{90}{100} = \frac{9}{10}, \quad v = 330 \text{ ms}^{-1}$

$\therefore \quad \frac{9}{10} = \frac{330 - v_0}{330}$

or $\quad 330 - v_0 = \frac{9}{10} \times 330 = 297$

$\therefore \quad v_0 = 330 - 297 = \mathbf{33 \text{ ms}^{-1}}.$

As the pitch of the sound appears to decrease, so the person is moving away from the observer.

EXAMPLE 90. *Two aeroplanes A and B are approaching each other and their velocities are 108 km h^{-1} and 144 km h^{-1} respectively. The frequency of a note emitted by A as heard by the passengers in B is 1170 Hz. Calculate the frequency of the note heard by the passenger in A. Velocity of sound = 350 ms^{-1}.*

Solution. Here aeroplane *A* (source) and aeroplane *B* (observer) both approach each other, so

$$v_s = +108 \text{ km h}^{-1} = +30 \text{ ms}^{-1},$$

$$v_0 = -144 \text{ km h}^{-1} = -40 \text{ ms}^{-1},$$

$$\nu' = 1170 \text{ Hz}, \quad v = 350 \text{ ms}^{-1}, \quad \nu = ?$$

As $\quad \nu' = \frac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad 1170 = \frac{350 + 40}{350 - 30} \times \nu$

$$\nu = \frac{1170 \times 320}{390} = \mathbf{960 \text{ Hz.}}$$

EXAMPLE 91. *A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15 rad s^{-1}. What is the lowest and highest frequency heard by a listener a long distance away at rest w.r.t. centre of the circle ? Can the apparent frequency be ever equal to the actual frequency ? Take $v = 330$ ms^{-1}.* **[MNREC 96 ; IIT 96]**

Solution. The situation is shown in Fig. 15.30.

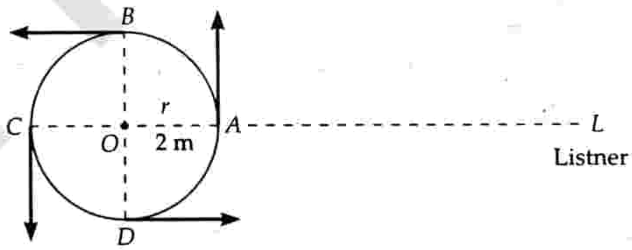


Fig. 15.30

Speed of source (whistle),

$$v_s = r\,\omega = 2 \times 15 = 30 \text{ ms}^{-1}$$

Actual frequency, $\nu = 540$ Hz,

Speed of sound, $\quad v = 330$ ms^{-1}

When the whistle (at position *B*) is moving away from the observer, the apparent frequency is lowest. It is given by

$$\nu' = \frac{v}{v + v_s} \times \nu = \frac{330}{330 + 30} \times 540 = 495 \text{ Hz}$$

When the whistle (at position *D*) is moving towards the observer, the apparent frequency is highest. It is given by

$$\nu'' = \frac{v}{v - v_s} \times \nu = \frac{330}{330 - 30} \times 540 = 594 \text{ Hz}$$

When the whistle is at *A* or *C*, its speed along $OL = v_s \cos 90° = 0$

$\therefore$ Apparent frequency = Actual frequency.

✖ Problems For Practice

1. A policeman blows a whistle with a frequency of 500 Hz. A car approaches him with a velocity of $15\ ms^{-1}$. Calculate the change in frequency as heard by the driver of the car as he passes the policeman. Speed of sound in air is $300\ ms^{-1}$. (**Ans.** 50 Hz)

2. Calculate the apparent frequency of the horn of a car approaching a stationary listener with a velocity of $12\ ms^{-1}$. The frequency of horn is 500 Hz. The speed of sound is $332\ ms^{-1}$. (**Ans.** 519 Hz)

3. A man standing near a railway line hears the whistle of an engine, which has a velocity of $20\ ms^{-1}$. What frequency does the man hear, when the engine is coming towards and going away from him, if the true frequency of the whistle is 1000 Hz ? Speed of sound in air = $340\ ms^{-1}$. (**Ans.** 1062.5 Hz, 944.4 Hz)

4. Two engines pass each other in opposite directions with a velocity of $60\ kmh^{-1}$ each. One of them is emitting a note of frequency 540. Calculate the frequencies heard in the other engine before and after they have passed each other. Given velocity of sound = $316.67\ ms^{-1}$. (**Ans.** 600 Hz, 486 Hz)

5. A train approaches a stationary observer, the velocity of train being 1/20 of the velocity of sound. A sharp blast is blown with the whistle of the engine at equal intervals of a second. Find the interval between the successive blasts as heard by the observer. (**Ans.** 19/20 s)

6. When an engine goes away from a stationary observer, the frequency of the engine appears $\frac{6}{7}$ times the real frequency. Calculate the speed of the engine. Speed of sound in air = $330\ ms^{-1}$. (**Ans.** $55\ ms^{-1}$)

7. A motor car is approaching towards a crossing with a velocity of $75\ kmh^{-1}$. The frequency of sound of its horn as heard by a policeman standing on the crossing is 260 Hz. What is the real frequency of the horn ? Speed of sound = $332\ ms^{-1}$. (**Ans.** 244 Hz)

8. Two cars are approaching each other on a straight road and moving with a velocity of $30\ kmh^{-1}$. If the sound produced in a car is of frequency 500 Hz, what will be the frequency of sound as heard by the person sitting in the other car ? When the two cars have crossed each other and are moving away from each other, what will be the frequency of sound as heard by the same person ? Speed of sound = $330\ ms^{-1}$. (**Ans.** 526 Hz, 475 Hz)

9. A source emitting sound of frequency 1000 Hz is moving towards an observer at speed $v_s = 0.90\ v$, (where v is the velocity of sound). What frequency will be heard by the observer ? (**Ans.** 10,000 Hz)

10. A policeman on duty detects a drop of 15% in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is $330\ ms^{-1}$, calculate the speed of the car. **[Central Schools 04 ; Chandigarh 08]** (**Ans.** $26.7\ ms^{-1}$)

11. The whistle of an engine moving at $30\ kmh^{-1}$ is heard by a motorist driving at $15\ kmh^{-1}$ and he estimated the pitch to be 500. What would be the actual pitch, if the two are approaching each other ? Velocity of sound = $1220\ kmh^{-1}$. (**Ans.** 481.8 Hz)

12. A car passing a check post gives sound of frequency 1000 cps. If the velocity of the car is $72\ kmh^{-1}$ and of sound is $350\ ms^{-1}$, find the change in apparent frequency as it crosses the post. (**Ans.** 114.66 Hz)

✖ Hints

1. When the car approaches the policeman,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{300 - (-15)}{300 - 0} \times 500 = 525 \text{ Hz}$$

When the car moves away from the police man,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{300 - 15}{300 - 0} \times 500 = 475 \text{ Hz}$$

Change in frequency $= \nu' - \nu'' = 525 - 475 =$ **50 Hz.**

3. Here $v_0 = 0, \quad v_s = 20\ ms^{-1}$,

$\nu = 1000$ Hz, $\quad v = 340\ ms^{-1}$

(*i*) When the engine approaches the man,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 20} \times 1000 = \mathbf{1062.5\ Hz.}$$

(*ii*) When the engine goes away from the man,

$$\nu'' = \frac{340 - 0}{340 - (-20)} \times 1000 = \mathbf{944.4\ Hz.}$$

4. Here $v_s = v_0 = 60$ km $h^{-1} = 16.67\ ms^{-1}$,

$\nu = 540$ Hz, $\quad v = 316.67\ ms^{-1}$

(*i*) *Before the two engines cross each other.*

$v_s = +16.67\ ms^{-1}, \quad v_0 = -16.67\ ms^{-1}$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{316.67 + 16.67}{316.67 - 16.67} \times 540$$

$=$ **600 Hz.**

(*ii*) *After the engines cross each other,*

$v_s = -16.67\ ms^{-1}, \quad v_0 = +16.67\ ms^{-1}$

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{316.67 - 16.67}{316.67 + 16.67} \times 540$$

$=$ **486 Hz.**

5. Let v be the velocity of sound.

Velocity of train (source of sound), $v_s = v/20$

Since the blast is blown at regular intervals of one second, therefore, the actual frequency of blasts,

$\nu = 1 s^{-1}$

The apparent frequency of blasts

$$\nu' = \frac{v}{v - v_s}\nu = \frac{v}{v - \frac{v}{20}} \times 1 = \frac{v}{\frac{19}{20}v} = \frac{20}{19}$$

The apparent interval between the two successive blasts,

$$t' = \frac{1}{\nu'} = \frac{1}{20/19} = \frac{\mathbf{19}}{\mathbf{20}}\ \mathbf{s.}$$

6. Here $v_0 = 0$, $\nu'/\nu = 6/7$, $v = 330\ \text{ms}^{-1}$, $v_s = ?$

As $\nu' = \frac{v - v_0}{v - v_s} \times \nu$

$\therefore \frac{\nu'}{\nu} = \frac{v - v_0}{v - v_s}$ or $\frac{6}{7} = \frac{330 - 0}{330 - v_s}$

or $v_s = \mathbf{-55\ ms^{-1}}$.

Negative sign shows engine goes away from the observer.

7. Here $v_s = 75\ \text{km h}^{-1} = 20.8\ \text{ms}^{-1}$, $v_0 = 0$,
$\nu' = 260\ \text{Hz}$, $\nu = ?$, $v = 332\ \text{ms}^{-1}$

As $\nu' = \frac{v - v_0}{v - v_s} \times \nu$ $\therefore$ $260 = \frac{332 - 0}{332 - 20.8} \times \nu$

or $\nu = \frac{260 \times 311.2}{332} = \mathbf{244\ Hz.}$

8. Here $v_s = v_0 = 30\ \text{kmh}^{-1} = 25/3\ \text{ms}^{-1}$,
$v = 330\ \text{ms}^{-1}$, $\nu = 500\ \text{Hz}$

(*i*) *When the two cars are approaching.*

$v_s = +25/3\ \text{ms}^{-1}$, $v_0 = -25/3\ \text{ms}^{-1}$

$$\nu' = \frac{330 + 25/3}{330 - 25/3} \times 500 = \frac{1015 \times 500}{965} = \mathbf{526\ Hz.}$$

(*ii*) *When the two cars are moving away from each other.*

$v_s = -25/3\ \text{ms}^{-1}$, $v_0 = +25/3\ \text{ms}^{-1}$

$$\nu'' = \frac{330 - 25/3}{330 + 25/3} \times 500 = \frac{965 \times 500}{1015} = \mathbf{475\ Hz.}$$

10. Before crossing, source is moving towards the stationary observer,

$$\nu' = \frac{v - 0}{v - v_s} \times \nu \qquad ...(1)$$

After crossing, source is moving away from the stationary observer,

$$\therefore \nu'' = \frac{v - 0}{v + v_s} \times \nu \qquad ...(2)$$

Dividing (2) by (1), $\frac{\nu''}{\nu'} = \frac{v - v_s}{v + v_s}$

As the pitch drops by 15%, so

$$\frac{\nu''}{\nu'} = \frac{85}{100} = \frac{17}{20}$$

Hence $\frac{17}{20} = \frac{330 - v_s}{330 + v_s}$

On solving, $v_s = \mathbf{26.7\ ms^{-1}}$.

15.25 CHARACTERISTICS OF MUSICAL SOUNDS

44. *Distinguish between music and noise.*

Music. The sound which has a pleasing sensation to the ears is called music. It is produced by regular and periodic vibrations, without any sudden change in loudness. Musical sound can be represented by a periodic wave function and can be splited into various harmonics.

Example. The sound produced by plucking the string of a sitar, by bowing the string of a voiline, sound from a tabla etc.

Noise. The sound which has non-pleasing or jarring effect on the ears is called noise. It is produced at irregular intervals and there is sudden change in loudness. The components of wave function have no definite regularities.

Example. The sound produced by an explosion, sound from a market, etc.

45. *Explain the characteristics of musical sounds. On what factors do they depend ?*

Characteristics of musical sounds. The *three* characteristics of musical sounds are (*i*) Loudness or intensity (*ii*) Pitch (*iii*) Quality or timbre.

(*i*) **Loudness.** *Loudness is the amount of energy crossing unit area around a point in one second.* Loudness depends on :

(*a*) Intensity which depends on amplitude and is proportional to the square of the amplitude.

(*b*) The surface area of the sounding body.

(*c*) Density of the medium.

(*d*) The presence of other resonant objects around the sounding body.

(*e*) The distance of the source from the listener.

(*f*) The motion of air.

(*ii*) **Pitch.** *Pitch is a sensation which helps a listener to distinguish between a high and a grave note. Pitch depends on frequency.*

The voice of a child or a lady is shriller than that of a man *i.e.*, the pitch of a lady's sound is higher than that of a man. The mosquito's sound is of high pitch and hence high frequency.

The pitch of a sound heard can change due to Doppler effect.

(*iii*) **Quality or timbre.** *Quality of sounds enables us to distinguish between two sounds of same pitch and loudness. It is due to the quality of sound that one can recognise one's friend without seeing him.* Quality of sound depends on the number of overtones present in it. It is due to different overtones present in musical instruments that we are able to recognise them by their sounds.

46. *What is meant by threshold of hearing ?*

Threshold of hearing. The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing. For a sound of frequency 10 kHz, the threshold of hearing is 10^{-12} Wm^{-2}.

47. *What is relation between loudness and intensity ?*

Relation between loudness and intensity. According to Weber-Fechner law, the loudness of a sound of intensity I is given by

$$L = \log_{10} \frac{I}{I_0}$$, where I_0 is the threshold of hearing.

48. *What is a musical scale ?*

Musical Scale. *A series of notes whose fundamental frequencies have definite ratios and which produce a pleasing effect on the ear when sounded in succession constitute a musical scale.* The simplest musical scale called the diatonic scale has eight notes comprising an octave. The frequency ratio of the eighth and the first note is 2 : 1. Conventionally, the fundamental frequency of the first note is taken to be 256 Hz and that of the last 512 Hz.

The frequencies of the intermediate notes with their Indian names are given below :

Symbol	*Indian Name*	*Frequency in the base 256 Hz*
C	Sa	256
D	Re	288
E	Ga	320
F	Ma	341.3
G	Pa	384
A	Dha	426.7
B	Ni	480
C_1	Sa	512

15.26 ACOUSTICS OF BUILDINGS

49. *What is meant by reverberation ? What is its cause ?*

Reverberation. *The persistence of audible sound after the source has ceased to emit sound is called reverberation.* When sound is produced in a hall or an auditorium, the sound waves suffer multiple reflections from the walls, ceiling and other materials present in the hall. The intensity of the sound heard is the combined effect of direct waves and reflected waves. Due to this, the sound persists for some time even after the source has stopped producing sound. This is the cause of reverberation.

50. *Define reverberation time. What is the optimum reverberation time for hearing a speech in a hall ?*

Reverberation time. *It is defined as the time which sound takes to fall in intensity to one millionth* (10^{-6}) *part of its original intensity after it was stopped.* Optimum time of reverberation depends upon the size of the hall, absorption material and the nature of programme to be heard. Reverberation time should neither be too high nor too low for quality of sound.

Subine formula for reverberation time of a hall is

$$T = \frac{0.16\ V}{\Sigma\ as}$$

Here V = Volume of the hall in cubic metre.

$\Sigma\ as = a_1s_1 + a_2s_2 + a_3s_3 + ...$

= Total absorption of the hall

$s_1, s_2, s_3, ...$ = Areas of the various surfaces in m^2.

$a_1, a_2, a_3, ...$ = Absorption coefficients of the the various surfaces.

Usually a reverberation period varying between 0.5 to 1.5 seconds is quite good enough in hearing depending upon the nature of auditorium and programme.

51. *How can reverberation time be controlled ?*

Methods for controlling reverberation time. The reverberation time can be controlled by following methods :

(*i*) Covering walls and doors with absorbent materials like asbestos, perforated card board, etc.
(*ii*) Providing open windows in the space.
(*iii*) Providing heavy curtains with folds and folding or opening some of the curtains.
(*iv*) Decorating the walls with pictures and maps.
(*v*) By increasing the number of audience.
(*vi*) By the floors which help in absorbing sound.

52. *What do you mean by acoustics ? Mention four important acoustical requirements of a building.*

Acoustics. *The branch of science which deals with the methods of production, reception and propagation of sound is called acoustics.*

Acoustical requirements of a building. These are as follows :

(*i*) The reverberation time must have the optimum value *i.e.*, it should neither be too low nor too high.
(*ii*) The total quality of music should not be altered.
(*iii*) There should be no echoes.
(*iv*) There should not be any undesirable focusing of sound due to reflections from the walls etc., and also there should not be any silence zones in the hall.

Very Short Answer Conceptual Problems

Problem 1. Is an oscillation a wave ? Give reason.

Solution. No, an oscillation is not a wave. The term wave implies the transfer of energy through successive vibrations of the particles of the medium. So the oscillations of a body do not constitute a wave.

Problem 2. A wave transmits momentum. Can it transfer angular momentum ?

Solution. A wave transmits momentum. It cannot transmit angular momentum. The transference of angular momentum means the action of a torque which causes rotatory motion.

Problem 3. Frequency is the most fundamental property of a wave. Why ?

Solution. When a wave travels from one medium to other, its wavelength as well as velocity may change. But frequency does not change. This is the reason that frequency is the fundamental property of a wave.

Problem 4. Which of the following is not a wave characteristic :

Reflection, refraction, interference, diffraction, polarisation, rectilinear propagation ?

Solution. Rectilinear propagation is not a wave characteristic.

Problem 5. How is energy transmitted in wave motion ?

Solution. The neighbouring oscillating parts of the medium are coupled together through elastic forces. During wave motion, a part of the medium is set into oscillation. This part hands over its motion to the next part of the medium and so on. This results in transmission of energy.

Problem 6. Name two important properties of a material medium responsible for the propagation of waves through it.

Solution. Properties of elasticity and inertia.

Problem 7. What is the source of electromagnetic waves ?

Solution. They are generated due to the changes of the electric and magnetic fields associated with the oscillating charges.

Problem 8. Why are the longitudinal waves also called pressure waves ?

Solution. Longitudinal waves travel in a medium as series of alternate compressions and rarefactions *i.e.*, they travel as variations in pressure and hence are called pressure waves.

Problem 9. What is a non-dispersive medium ? Give an example.

Solution. A medium in which the speed of a wave is independent of its frequency is called a non-dispersive medium. For example, air is a non-dispersive medium for sound waves.

Problem 10. Can transverse waves be produced in air ?

Solution. No. Transverse waves travel in the form of crests and troughs and so involve change in shape. So the transverse waves can be produced in a medium which has elasticity of shape. As air has no elasticity of shape, hence transverse waves cannot be produced in it.

Problem 11. Do displacement, particle velocity and pressure variation in a longitudinal wave vary with the same phase ?

Solution. No. The particle velocity is $\pi/2$ out of phase with the displacement and the pressure variation is out of by phase by π with the displacement.

Problem 12. How can we distinguish experimentally between longitudinal and transverse waves ?

Solution. We can distinguish between longitudinal and transverse waves by performing polarization experiments. Transverse waves can be polarised while longitudinal waves cannot be polarised.

Problem 13. What is the direction of oscillations of the particle of the medium through which (*i*) a transverse and (*ii*) a longitudinal wave is propagating ?

Solution. (*i*) When a transverse wave propagates through a medium, its particles oscillate perpendicular to the direction of propagation of the wave.

(*ii*) When a longitudinal wave propagates through a medium, its particles oscillate along the direction of propagation of the wave.

Problem 14. What is the difference between wave velocity and particle velocity ?

Solution. The wave velocity (or phase velocity) is constant for a given medium and is given by $v = \nu\lambda$ while the particle velocity changes harmonically with time. It is maximum at the mean position and zero at the extreme position.

Problem 15. We always see lightning before we hear thundering. Why ?

Solution. The speed of light (3×10^8 ms^{-1}) is much larger than the speed of sound (~ 340 ms^{-1}). Consequently, the flash of light reaches us much earlier than the sound of thunder.

Problem 16. What does cause the rolling sound of thunder ?

Solution. The multiple reflections of sound of lightning produce the rolling sound of thunder.

Problem 17. Two astronauts on the surface of the moon cannot talk to each other. Why ?

Solution. Sound waves require material medium for their propagation. As there is no atmosphere on the moon, hence the sound wave cannot propagate on the moon.

Problem 18. Why explosions on other planets cannot be heard on the earth ?

Solution. Since no material medium is present in the space between the planets and the earth, so the sound of explosions on other planets cannot propagate upto the earth.

Problem 19. When a stone is thrown on the surface of water, a wave travels out. From where does the energy come ?

Solution. The energy of the surface wave spreading on the surface of water comes from the kinetic energy of the stone shared by the water molecules on which it falls.

Problem 20. Why does sound travel faster in solids than in gases ? *[Himachal 03]*

Solution. The speed of sound through any medium of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

The value of the ratio κ / ρ is much higher for solids than that for gases. That is why sound travels faster in solids than in gases.

Problem 21. How is it possible to detect the approaching of a distant train by placing the ear very close to the railway line ?

Solution. Sound waves travel much faster in solids than that in air. Moreover, due to high elasticity of solids, sound waves do not die out in solids as soon as in air. This makes possible to detect the sound of a distant approaching train by placing the ear very close to the railway line.

Problem 22. If a person places his ear to one end of a long iron pipeline, he can distinctly hear two sounds when a workman hammers the other end of the pipeline. How ?

Solution. Sound travels sixteen times faster in iron than in air. So the person hears two sounds, the first one travelling through the iron pipeline and the second travelling through air.

Problem 23. Ocean waves hitting a beach are always found to be nearly normal to the shore. Why ?

Solution. Ocean waves are transverse in nature and spread out in the from of concentric circles. When these waves reach the beach shore, their radius of curvature becomes so large that they can be treated as plane waves. Hence the ocean waves hit the beach nearly normal to the shore.

Problem 24. In which medium, do the sound waves travel faster, solids, liquids or gases ? Give reason.

Solution. Sound waves travel in solids with highest speed. This is because the coefficient of elasticity of solids is much greater than coefficient of elasticity of liquids and gases.

Problem 25. Sound travels faster on a rainy day than on a dry day. Why ?

Solution. The amount of water vapours present in the atmosphere is much higher on a rainy day than on a dry day. As the water vapours are lighter than dry air, hence density of wet air becomes less than that of dry air. Now, because the speed of sound is inversely proportional to the square root of the density, hence sound travels faster on a rainy day than on a dry day.

Problem 26. If the pressure of a gas at constant temperature is increased four times, how the velocity of sound in the gas will be affected ? *[Central Schools 11]*

Solution. $v = \sqrt{\frac{\gamma P}{\rho}}$, any increase in '$P$' produces corresponding increase in 'ρ' so that $\frac{P}{\rho}$ = constant. Hence the velocity of sound in a gas is independent of pressure.

Problem 27. Explain why sound travels faster in warm air than cool air ? *[Himachal 05C, 06]*

Solution. Velocity of sound is directly proportional to the square root of the temperature of the air

i.e., $v \propto \sqrt{T}$.

As temperature of the warm air is more than that of the cool air, so sound travels faster in warm air than in cool air.

Problem 28. What is the nature of thermal changes in air when sound propagates through air ?

Solution. The sound wave travels through air under adiabatic conditions.

Problem 29. What characteristics of a medium determine the speed of sound waves through it ?

Solution. The speed of sound waves in a medium is determined by (*i*) elasticity and (*ii*) density of the medium.

Problem 30. If we set our watch by the sound of a distant siren, will it go slow or fast ?

Solution. The speed of sound in air has a finite value of nearly 350 ms^{-1}. The sound of siren will take a finite time to reach us. Hence the watch set according to the sound of distant siren will go slow by as much time as that taken by sound to reach us.

Problem 31. What will be the velocity of sound in a perfectly rigid rod ? Give reason.

Solution. The velocity of sound in a perfectly rigid rod will be infinite. This is because the value of Young's modulus of elasticity is infinite for a perfectly rigid rod.

Problem 32. Sound is simultaneously produced at the ends of the two strings of the same length, one of rubber and the other of steel. In which string will the sound reach the other end earlier and why ?

Solution. Speed of sound in a string, $v = \sqrt{\frac{Y}{\rho}}$.

As the value of Y / ρ is larger for steel than for rubber, so sound will reach the other end earlier in the case of steel string.

Problem 33. The speed of sound does not depend upon its frequency. Give an example in support of this statement.

Solution. If sounds are produced by different musical instruments simultaneously, then all these sounds are heard by the ear at the same time.

Problem 34. The speed of sound in moist air is greater than that in dry air, why ? Will the speed of sound in moist hydrogen be greater than that in dry hydrogen ?

Solution. The density of water vapour is less than that of air. So the density of air mixed with water vapour (moist air) is less than that of dry air. Hence the speed of sound in moist air is greater than that in dry air ($v \propto 1/\sqrt{\rho}$). However, the density of water vapour is more than that of hydrogen, so the density of moist hydrogen is more than that of dry hydrogen. Consequently, the speed of sound in moist hydrogen is less than that in dry hydrogen.

Problem 35. The velocity of sound in a tube containing air at 27° C and a pressure 76 cm of mercury is 330 ms^{-1}. What will be the velocity of sound when pressure is increased to 100 cm of mercury and the temperature is kept constant ?

Solution. At a given temperature, the velocity of sound in a gas is independent of pressure. Hence the velocity of sound in the tube will remain 330 ms^{-1}.

Problem 36. The shape of a pulse gets distorted as it passes through a dispersive medium. Why ?

Solution. When a pulse passes through a dispersive medium, the wavelength of the wave changes. Consequently, the shape of the pulse changes *i.e.*, it gets distorted.

Problem 37. If an explosion takes place at the bottom of a lake, will the shock waves in water be longitudinal or transverse ?

Solution. An explosion in a lake produces shock waves thereby creating enormous increase in pressure in the medium (water). A shock wave is thus a longitudinal wave travelling at a speed which is greater than that of a longitudinal wave of ordinary intensity.

Problem 38. Does the sound of a bomb explosion travel faster than the sound produced by a humming bee ?

Solution. The velocity of sound in a medium does not depend upon its loudness, pitch or quality. Thus the sound of bomb explosion and of a humming bee, even though having entirely different characteristics, travel with the same speed.

Problem 39. If a balloon is filled with CO_2 gas, then how will it behave for sound as a lens ? What happens if CO_2 gas is replaced H_2 gas ?

Solution. The velocity of sound in CO_2 is less than that in air. Hence a balloon filled with CO_2 will behave like a convex lens. But the velocity of sound in H_2 is greater than that in air, so a balloon filled with H_2 will behave like a concave lens.

Problem 40. Sound can be heard over longer distances on a rainy day. Why ? *[Himachal 06]*

Solution. On a rainy day, the air contains a larger amount of water vapour. This decreases the density of air. As a result, the sound travels faster in air and can be heard over longer distances.

Problem 41. Why sound can be heard more distinctly at a greater distance over water surface ?

Solution. Sound waves are almost totally reflected from the air-water interface because the critical angle for this interface is only 14°. Consequently, the listener receives more sound energy. Hence sound is heard more distinctly.

Problem 42. What is a periodic wave function ?

Solution. A wave function $y(x, t)$ of position and time which satisfies the following periodicity conditions is called periodic wave function :

(*i*) $y(x + m\lambda, t) = y(x, t)$ (*ii*) $y(x, t + nT) = y(x, t)$.

Problem 43. What is a harmonic wave function ?

Solution. A periodic wave function, whose functional form is sine or cosine is called harmonic wave function.

Problem 44. A heavy uniform rope is held vertically and is tensioned by clamping it to a rigid support at the lower end. A wave of certain frequency is set up at the lower end. Will the wave travel up the rope at the same speed ?

Solution. No. Due to the weight of the rope, the tension increases along the rope from the lower end to the upper end. Hence the wave will travel up the rope with an increasing velocity.

Problem 45. Sometimes, in a stringed instrument, a thick wire is wrapped by a thin wire. Why ?

Solution. This increases mass per unit length and hence helps in obtaining a desired low frequency which would otherwise require a string of inconveniently large length.

Problem 46. Why are stationary waves called so ? *[Himachal 04, 05, 05C]*

Solution. In a stationary wave, the particles of the medium vibrate about their mean positions, but disturbance does not travel in any direction.

Problem 47. When are stationary waves produced ?

Solution. Stationary waves are produced when two identical waves travelling in opposite directions through a medium superpose each other.

Problem 48. Under what condition does a sudden phase reversal of waves on reflection take place ?

Solution. On reflection from a denser medium, a wave suffers a sudden phase reversal.

Problem 49. A light wave is reflected from a mirror and the incident and the reflected waves superpose to form stationary waves. Explain why nodes and antinodes are not observed, similar to that found in case of sound waves.

Solution. The wavelength of light is of the order of 10^{-7} metre, hence the distance between the nodes or antinodes will also be of the same order. It is not possible to resolve this distance by eye or by an ordinary optical instrument.

Problem 50. What is difference between a tone and a note ?

Solution. A sound of single frequency is called a tone. A combination of tones of different frequencies is called a note.

Problem 51. Why is the note produced by a open organ pipe sweeter than that produced by the closed organ pipe ?

Solution. The note produced by open organ pipe consists of both odd and even harmonics but the note produced by closed organ pipe consists of only the odd harmonics. Due to presence of larger number of overtones or harmonics, the note produced by the open organ pipe is sweeter.

Problem 52. Why are there so many holes in a flute ?

Solution. The flute is basically an open organ pipe. The location of the open end can be changed by keeping the one hole open and closing the other holes. Thus the frequency of the note produced by the flute can be changed.

Problem 53. Why does the pitch of a note produced by a wooden open end pipe becomes sharper when the temperature rises ?

Solution. With rise in temperature, the velocity of sound increases. The fundamental frequency of an open organ pipe is given by ($\nu_1 = v/2l$). Hence with an increase in the value of v, ν_1 increases and the pitch of the note becomes sharper.

Problem 54. When we start filling an empty bucket with water, the pitch of sound produced goes on changing. Why ?

Solution. A bucket may be regarded as a pipe closed at one end. It produces a note of frequency,

$$\nu = \frac{v}{4L}$$

where v is the velocity of sound in air and L is the length of air column, which is equal to depth of water level from the open end. As the bucket is filled with water, the value of L decreases. Consequently, the frequency and hence the pitch of the sound produced goes on changing.

Problem 55. A vessel is placed below a water-tap. We can estimate the height of water level reached in the vessel from a distance simply by listening the sound. How ?

Solution. The frequency of the note produced by an air column is inversely proportional to its length. As the level of water in the vessel rises, the length of the air column above it decreases. It produces sound of decreasing frequency *i.e.,* the sound becomes shriller. From the extent of shrillness of sound, we can estimate the height upto which the vessel has been filled with water.

Problem 56. If oil of density higher than the density of water is used in a resonance tube, how will the frequency change ?

Solution. The frequency of vibration depends on the length of the air column. The liquid surface only causes the reflection of water. Hence frequency does not change if oil of density higher than that of water is used in the resonance tube.

Problem 57. A vibrating string is heated to a higher temperature. What happens to the pitch of the note produced by it ?

Solution. When a string is heated to a higher temperature, its length increases and its frequency of vibration decreases ($\nu \propto 1/L$). Hence the pitch of the note produced by it decreases.

Problem 58. Why are strings of different thicknesses and materials are used in a sitar or a violin ?

Solution. Fundamental frequency of a stretched string,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

When we use strings of different thicknesses and materials, they have different values of mass per unit length (m). So the strings will produce notes of different frequencies.

Problem 59. A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of same length. Why ?

Solution. As the tuning fork of frequency ν is in resonance with a closed pipe of length L, so

$$\nu = \frac{v}{4L}$$

But an open pipe of length L produces a frequency of $v/2L$. Hence it cannot be in resonance with the tuning fork of frequency $\nu\ (= v/4L)$.

Problem 60. Why does the pitch of a note produced by a wooden open end organ pipe become shaper when the temperature increases ?

Solution. The fundamental frequency of an open end organ pipe is given by

$$\nu = \frac{v}{2L}$$

As the temperature increases, the velocity of sound (v) increases. The frequency ν increases. The pitch of the note produced becomes sharper.

Problem 61. Two organ pipes of same length open at both ends produce sound of different frequencies if their radii are different. Why ?

Solution. The fundamental frequency of an open organ pipe is given by

$$\nu = \frac{v}{2(L + 0.3\,D)}$$

Here D is the diameter of the tube and $0.3\,D$ is the *end correction*. Obviously, two organ pipes of same length but of different radii will produce sounds of different frequencies.

Problem 62. How does the frequency of a tunning fork change, when the temperature is increased ? *[AIEEE 02]*

Solution. As the temperature increases, the length of the prong of the tuning fork increases. This increases the wavelength of the stationary waves set up in the tuning fork. As frequency, $\nu \propto 1/\lambda$, so frequency of the tuning fork decreases.

Problem 63. How does the frequency of a vibrating wire change, when the attached load is immersed in water ?

Solution. When the load is immersed in water, its apparent weight decreases. So the tension in the string decreases. As $\nu \propto \sqrt{T}$, the frequency of the vibrating wire decreases.

Problem 64. What points of the stretched string between the fixed points must be plucked and touched to excite its second harmonic ?

Solution. In the second harmonic, there is a node at the centre and two antinodes at $1/4$ and $3/4$ points. To excite second harmonic, the wire should be plucked at its one-fourth length and touched at its centre.

Problem 65. What is the function of the wooden box in the sonometer ? Does it increase or decrease the duration of emission ?

Solution. The function of the wooden box in a sonometer is to increase the sound intensity by its forced vibrations. It decreases the duration of emission of sound energy.

Problem 66. Why is a sonometer box provided with holes ?

Solution. The holes in the walls of the sonometer box keep the inside air in contact with the outside air. When the sonometer wire vibrates, the vibrations are handed over from the bridges to the upper surface of the sound-board and the air inside it. Consequently, the outside air also begins to vibrate and a loud sound is heard.

Problem 67. The beats are not heard, if the difference in frequencies of the two sounding notes is more than 10. Why ? *[Himachal 06]*

Solution. If the difference in frequencies of the two waves is more than 10, we shall hear more than 10 beats per second. Due to persistence of hearing, our ear is not able to distinguish between two sounds as separate if the time interval between them is less than (1/10)th of a second. Hence beats heard will not be distinct if the number of beats produced per second is more than 10.

Problem 68. Why do we not hear beats due to sound waves produced by the violins in the violin-section of an orchestra ?

Solution. All the violins in the violin section of an orchestra are tuned to the same frequency. Since there is no difference in the frequencies of these violins, no beats are heard.

Problem 69. As in sound, can beats be observed by two light-sources ?

Solution. No, to observe beats by two light-sources the phase difference between the sources should change regularly. In light sources, however this change occurs at random, because the light-source consists of a large number of atoms and each atom emits wave independently.

Problem 70. Is it necessary for beat production that the two waves must have exactly equal amplitudes ?

Solution. It is not at all necessary that the amplitudes of two waves producing beats should be equal. It is only when we wish to get zero sound at minima that the two amplitudes should be equal. However, the beats become more clear as the amplitudes of two waves approach each other.

Problem 71. If two sound waves of frequencies 500 Hz and 550 Hz superpose, will they produce beats ? Would you hear the beats ?

Solution. Yes, the sound waves will produce 50 beats every second. But due to persistence of hearing, we would not be able to hear these beats. We would hear a continuous sound of frequency 50 Hz, called beat tone.

Problem 72. Can we hear beats when sounds from two different sources are heard together ?

Solution. Yes. Though the two sources are not coherent, they can produce beats.

Problem 73. Does Doppler's effect apply to only sound waves ?

Solution. No, it applies to light waves also.

Problem 74. What physical change occurs when a source of sound moves and the listener is stationary ?

Solution. Wavelength of sound waves changes.

Problem 75. What physical change occurs when the source of sound is stationary but the listener moves ?

Solution. The number of sound waves received by the listener changes.

Problem 76. Will there be Doppler effect, when the direction of motion of the source or observer is perpendicular to the direction of propagation of sound ?

Solution. No, there is no Doppler effect, when the source or observer moves perpendicular to the direction of propagation of sound.

Problem 77. A person riding on a merry-go-round emits a sound wave of a certain frequency. Will the person at the centre observe Doppler effect ?

Solution. No, because the source moves perpendicular to the line joining the source and the observer.

Problem 78. Will there be any Doppler effect, if both the sound and the listener are moving with the same velocity and in the same direction ?

Solution. The apparent frequency is given by

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

As $v_0 = v_s$, therefore, $\nu' = \nu$ *i.e.* there is no Doppler effect.

Problem 79. What is an echo ? What should be the minimum distance between the source of sound and the reflector for hearing a distinct echo ?

Solution. *Echo is the phenomenon of repetition of sound due to its reflection from the surface of a large obstacle.*

If s be the distance between the source and reflector, v the velocity of sound and t be the total time taken by sound to reach the listener after the reflection, then

$$2s = vt \quad \text{or} \quad s = \frac{vt}{2} = \frac{340}{2} \times \frac{1}{10} = 17 \text{ m.}$$

Problem 80. Explain why we cannot hear an echo in a small room ?

Solution. For an echo of a simple sound to be heard, the minimum distance between the speaker and the walls should be 17 m. As the length of a room is generally less than 17 m, so we do not hear an echo.

Problem 81. What do you mean by reverberation ? What is reverberation time ?

Solution. The phenomenon of persistence or prolongation of sound after the source has stopped emitting sound is called reverberation. The time for which the sound persists until it becomes inaudible is called the reverberation time.

Problem 82. What is the difference between an echo and a reverberation ?

Solution. An echo is produced when sound reflected from a distant obstacle comes back after an interval of 1/10 second or more. In an echo, the original and reflected sounds are heard separately. Reverberation, on the other hand, consists of successive reflections which follow each other so quickly that they cannot produce separate echoes.

Problem 83. The reverberation time is larger for an empty hall than in a crowded hall. Why ?

Solution. In a crowded hall, the absorption of sound waves is much more than in an empty hall because reverberation depends upon the total absorbing material in the hall.

Problem 84. Thick and long curtains are preferred in a big hall. Why ?

Solution. A big hall has large reverberation time due to which different syllables are not heard distinctly. By making use of thick and long curtains, which have large absorption coefficient, reverberation time can be suitably decreased.

Problem 85. An organ pipe emits a fundamental note of frequency 128 Hz. On blowing into it more strongly it produces the first overtone of frequency 384 Hz. What is the type of pipe – closed or open ? *[Delhi 96]*

Solution. The organ pipe must be a closed organ pipe, because the frequency of the first overtone is 3 times the fundamental frequency.

Problem 86. What are infrasonics and ultrasonics ?

Solution. Frequencies below 20 Hz are called infrasonics. Frequencies above 20,000 Hz are called ultrasonics.

Problem 87. How do we identify our friend from his voice while sitting in a dark room ?

Solution. On the basis of quality of sound.

Problem 88. What determines the quality of sound ?

Solution. Quality of sound is determined by the number of harmonic components present in the sound.

Problem 89. A violin note and a sitar note may have the same frequency and yet we can distinguish between the two notes. Explain, why it is so.

Solution. This is due to the fact that overtones produced by the two sources may be different. In other words the quality of sound produced by two instruments of same fundamental frequency is different.

Problem 90. What do you understand by the fidelity of an instrument ?

Solution. The property of a device to reproduce the original sound in all its details is called its fidelity.

Problem 91. What is the factor on which pitch of a sound depends ?

Solution. The pitch of a sound depends on its frequency.

Problem 92. Where will a man hear a louder sound – at the node or at the antinode in case of a stationary wave ?

Solution. The sound is heard due to variation in pressure, which is given by

$$\Delta P = -\text{ Elasticity} \times \text{Strain}$$

At the antinodes, amplitude is maximum but strain is minimum. At the nodes, the amplitude is minimum but strain is maximum. So variation in pressure is maximum at the nodes. Hence a loud sound is heard at node not at antinode.

Short Answer Conceptual Problems

Problem 1. Given below are some examples of wave motion. State in each case, if the wave motion is transverse, longitudinal or a combination of both :

(i) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.

(ii) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(iii) Waves produced by a motor boat sailing in water.

(iv) Light waves travelling from the sun to the earth.

(v) Ultrasonic waves in air produced by a vibrating quartz crystal. *[NCERT]*

Solution. (i) ***Transverse wave motion,*** because the vibrations of particles (kinks) of the spring are at right angles to the direction of wave propagation.

(ii) ***Longitudinal wave motion,*** because the molecules of the liquid vibrate to and fro about their mean position along the direction of propagation of the wave.

(iii) ***Combination of longitudinal and transverse waves,*** because the propeller of the molar boat cuts the water surface laterally and also pushes it in backward direction.

(iv) ***Transverse wave motion,*** because the light waves are electromagnetic waves in which electric and magnetic fields vibrate in the direction at right angle to each other and also to the direction of propagation of the wave.

(v) ***Ultrasonic waves produced by a quartz crystal in air are longitudinal*** because the molecules of air vibrate to and fro about their mean positions along the direction of propagation of wave due to vibrations of quartz crystal.

Problem 2. Why is the sound produced in air not heard by a person deep inside the water ?

Solution. The speed of sound in water is nearly four times the speed of sound in air. From Snell's law of refraction,

$$\mu = \frac{\sin i}{\sin r} = \frac{v_a}{v_w} = \frac{1}{4} = 0.25$$

For refraction, $r_{max} = 90°$, so $(\sin i)_{max} = 0.25$. Hence $i_{max} = 14°$. Consequently, most of the sound produced in air and incident at $\angle i > 14°$ gets reflected back in air and very small amount is refracted into water. Hence a person deep inside water cannot hear the sound produced in air.

Problem 3. In summer, the sound of a siren is heard louder in the night than in the day to a person on the earth. Why ?

Solution. During the day, the temperature of the earth is maximum near the ground and it progressively decreases upwards. The velocity of sound is maximum near the ground and decreases upwards ($v \propto \sqrt{T}$). The vertical plane wavefronts produced by the siren continuously bend upwards, so the sound waves curl upwards during the day. The temperature conditions are reversed at night. The sound waves curl downwards, making the sound of siren louder on the earth.

Problem 4. If two waves of same frequency but of different amplitudes travelling in opposite directions through a medium superpose upon each other, will they form stationary wave ? Is energy transferred ? Are there any nodes ?

Solution. Yes, the given waves superpose to form stationary waves of the form shown in Fig. 15.31. No energy is transferred. There are no nodes but there are positions of minimum amplitude.

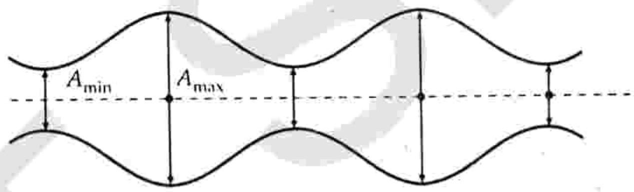


Fig. 15.31

Problem 5. What are overtones and harmonics ? The presence of which makes a sound musical ?

Or

Distinguish between harmonics and overtones.

[Himachal 06]

Solution. A vibrating system such as a stretched string, an air column, etc., can vibrate with a number of frequencies. The mode with lowest frequency is called *fundamental mode* and others are called *overtones*. When the frequencies of overtones are exact multiples of the fundamental frequency, they are called *harmonics*. The fundamental itself is called *first harmonic*. The presence of the harmonics makes a sound musical.

Problem 6. All harmonics are overtones but all overtones are not harmonics. How ?

Solution. The overtones with frequencies which are integral multiples of the fundamental are called harmonics. Hence all harmonics are overtones. But overtones which are non-integral multiples of the fundamental are not harmonics.

Problem 7. The fundamental frequency of a source of sound is 200 Hz and the source produces all the harmonics. State, with reasons, with which of the following frequencies this source will resonate : 150, 200, 300 and 600 Hz ?

Solution. The source will produce harmonics of frequencies 200 Hz, 400 Hz, 600 Hz, 800 Hz, etc. Clearly, the source will resonate with frequencies of 200 Hz and 600 Hz.

Problem 8. An organ pipe is in resonance with a tuning fork. What change will have to be done in the length L to maintain the resonance, if (*i*) the temperature increases, (*ii*) hydrogen is filled in place of air and (*iii*) pressure becomes higher ?

Solution. Suppose the organ pipe has both ends open. Then its fundamental frequency of vibration will be

$$\nu = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}}$$

(*i*) As the temperature increases, the velocity of sound increases, so to fix ν constant, the value of L must be increased.

(*ii*) If hydrogen is filled in the pipe in place of air, the value of density ρ decreases. To keep ν constant, the value of L must be increased.

(*iii*) The pressure has no effect on the velocity of sound. It does not affect resonance.

Problem 9. Fig. 15.32 shows two vibrating modes of an air column. Find the ratio of frequencies of the two modes.

Solution. Let L be the length of the air column.

For mode (*a*),

$$L = 3\lambda/4 \quad \text{or} \quad \lambda = 4L/3$$

$\therefore$ Frequency,

$$\nu_1 = \frac{v}{\lambda} = \frac{3v}{4L}$$

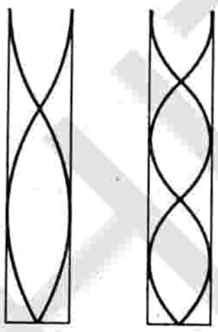
For mode (*b*),

$$L = 5\lambda/4 \quad \text{or} \quad \lambda = 4L/5$$

$\therefore$ Frequency,

$$\nu_2 = \frac{v}{\lambda} = \frac{5v}{4L}$$

Hence $\nu_1 : \nu_2 = 3 : 5$.



(*a*) (*b*)

Fig. 15.32

Problem 10. Two progressive sound waves each of frequency 170 Hz and travelling in opposite directions in air superpose to produce stationary waves. The speed of sound in air is 340 ms^{-1}. What is the separation between (*i*) two successive nodes, (*ii*) two successive antinodes and (*iii*) a node and its nearest antinode ?

Solution. Wavelength of sound,

$$\lambda = \frac{v}{\nu} = \frac{340}{170} = 2.0 \text{ m}$$

(*i*) Distance between two successive nodes

$$= \lambda/2 = \mathbf{1.0\ m.}$$

(*ii*) Distance between two successive antinodes

$$= \lambda/2 = \mathbf{1.0\ m.}$$

(*iii*) Distance between a node and its nearest antinode

$$= \lambda/4 = \mathbf{0.5\ m.}$$

Problem 11. A sonometer wire resonates with a tuning fork. If the length of the wire between the bridges is made twice even then it can resonate with the same tuning fork. How ?

Solution. Initially, the fundamental frequency of the sonometer wire is equal to the frequency ν of the tuning fork. When the length of the wire is made twice, its fundamental frequency becomes $\nu/2$. But it now produces a first overtone of frequency ν, vibrating in two segments. Hence the wire can still vibrate in resonance with the tuning fork.

Problem 12. Why does a tuning fork have two prongs ? Would the tuning fork be of any use, if one of the prongs is cut off ?

Solution. When a tuning fork is set into vibrations, its two prongs vibrate in opposite phases. However, the centre of mass of the tuning fork, which lies at the midpoint of the bend, remains at rest. Hence by holding its stem in the hand, a tuning fork can be set into vibrations and no external force is required to maintain its vibrations.

If one of the prongs is cut off, the oscillations of the tuning fork will soon die out and can be maintained only with the help of an external periodic force.

Problem 13. Why is a tuning fork used as a standard oscillator ? On what factors does the pitch of a tuning fork depend ?

Solution. When a tuning fork is struck lightly against a rubber pad, it produces only fundamental tone. If it is struck forcefully, it produces overtones which soon die out. So a tuning fork can be used as a source of standard frequency.

Factors on which the pitch of a tuning fork depends :

(*i*) It is inversely proportional to the square of the length of its prongs. Thus

$$\nu \propto \frac{1}{l^2}$$

(*ii*) It is directly proportional to the thickness of the fork.

$$\nu \propto b$$

(*iii*) It is directly proportional to the square root of the Young's modulus of elasticity of its material.

$$\nu \propto \sqrt{Y}$$

(*iv*) It is inversely proportional to the square root of the density of its material.

$$\nu \propto \frac{1}{\sqrt{\rho}}.$$

Hence low-frequency tuning forks are long and thin while high-frequency tuning forks are short and thick.

Problem 14. A sitar wire and a tabla, when sounded together, produce 5 beats per second. What can be concluded from this ? If the tabla membrane is tightened, will the beat rate increase or decrease ?

Solution. If ν_1 and ν_2 are the frequencies of sitar wire and the tabla membrane, then

$$\nu_1 \sim \nu_2 = 5$$

If the tabla membrane is tightened *i.e.*, tension is increased, the frequency (ν_2) of the sound produced by the tabla will increase. If $\nu_1 > \nu_2$, the beat frequency will decrease. And if $\nu_1 < \nu_2$, the beat frequency will increase.

Problem 15. Doppler effect is asymmetric in sound whereas in case of light it is symmetric. Explain.

Solution. Sound waves require a material medium for their propagation. So the observed frequency of sound when the source moves towards the observer is different from the case when the observer moves towards the source with the same relative velocity. We say that the Doppler effect in sound is asymmetric. On the other hand, no material medium is required for propagation of light waves. So the apparent frequency is same whether the source moves towards the observer or the observer moves towards the source. We say that the Doppler effect in light is symmetric.

Problem 16. Distinguish between transverse and longitudinal waves. *[Himachal 09]*

Solution.

Transverse waves	Longitudinal waves
1. The vibrations of the particles of the medium are perpendicular to the direction of propagation of the wave.	The vibrations of the particles of the medium are parallel to the direction of propagation of the wave.
2. In transverse waves, alternate crests and troughs are formed.	In longitudinal waves, alternate zones of compression and rarefaction are formed.
3. These waves may be formed in solids and over liquid surfaces.	These waves may be formed in solids, liquids and gases.
4. These waves do not involve changes of pressure and density of the medium.	These waves involve changes of pressure and density of the medium.
5. These waves can be polarised.	These waves cannot be polarised.

Problem 17. What is red shift ? What does it indicate ?

Solution. The spectral lines received from the distant stars and galaxies are found to be shifted towards the higher wavelength side *i.e.*, towards the red end of the visible spectrum. This shift in wavelength is called red shift. This indicates that stars and galaxies are receding away from us or the universe is expanding.

Problem 18. An incident wave is represented by $y(x, t) = 20 \sin(2x - 4t)$. Write the expression for reflected wave :

(*i*) from a rigid boundary.

(*ii*) from an open boundary. *[Central Schools 03, 11]*

Solution. (*i*) The wave reflected from a rigid boundary is

$$y(x, t) = -20 \sin(-2x - 4t) = \mathbf{20 \sin (2x + 4t)}.$$

(*ii*) The wave reflected from an open boundary is

$$y(x, t) = 20 \sin(-2x - 4t) = \mathbf{-20 \sin (2x + 4t)}.$$

Problem 19. State the principle of superposition of waves. Distinguish between conditions for the production of stationary waves and beats. *[Delhi 03C]*

Solution. The *principle of superposition states* that when a number of waves travel through a medium simultaneously, the resultant displacement of any, particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y = y_1 + y_2 + y_3 + \ldots + y_n$$

(*i*) When two waves of same frequency moving with the same speed in the opposite directions in medium superpose an each other, they produce *stationary waves.*

(*ii*) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each, they produce *beats.*

Problem 20. Differentiate between Stationary waves and Progressive waves. *[Himachal 05 ; Delhi 11]*

Solution.

Stationary Waves	Progresssive Waves
(*i*) The disturbance remains confined to a particular region, and there is no onward motion.	The disturbance travels forward, being handed over from one particle to the neighbouring particle.
(*ii*) There is no transfer of energy in the medium.	Energy is transferred in the medium along the waves.
(*iii*) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.	The amplitude of vibration of each particle is same.
(*iv*) The particles of the medium at nodes are permanently at rest.	No particle of the medium is permanently at rest.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *The length of a string tied to two rigid supports is 40 cm. What is the maximum length (wavelength in cm) of the stationary wave produced on it?* **[AIEEE 02]**

Solution. When the string vibrates in one segment,

$$L = \frac{\lambda}{2}$$

or $\lambda = 2L = 2 \times 40$ cm $= \mathbf{80\ cm.}$

Problem 2. *Tube A has both ends open, while B has one end closed. Otherwise the two tubes are identical. What is the ratio of fundamental frequency of the tubes A and B?* **[AIEEE 02]**

Solution. The fundamental frequency for tube A with both ends open is

$$\nu_A = \frac{v}{2L}$$

The fundamental frequency for tube B with one end closed is

$$\nu_B = \frac{v}{4L}$$

$$\therefore \quad \frac{\nu_A}{\nu_B} = \frac{v/2L}{v/4L} = \mathbf{2.}$$

Problem 3. *An open pipe is in second harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Find the value of n. How are f_1 and f_2 related to each other?* **[IIT 05]**

Solution. With both ends of the pipe open, the frequency of second harmonic is

$$f_1 = 2 \cdot \frac{v}{2L} = \frac{v}{L}$$

With one end of the pipe closed, the frequency of n^{th} harmonic is

$$f_n = n\frac{v}{4L},$$

where n is an odd integer.

Clearly, f_n will be just greater than f, when $n = 5$.

Hence $\quad f_2 = 5\frac{v}{4L} = \frac{5}{4} f_1.$

Problem 4. *A string is stretched between two fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. What is the lowest resonant frequency for this string?* **[AIEEE 06]**

Solution. Here $p\frac{v}{2l} = 315\,\text{Hz}$

and $\quad (p+1)\frac{v}{2l} = 420\ \text{Hz}$

Hence the lowest resonant frequency,

$$\nu = \frac{v}{2l} = 420 - 315 = \mathbf{105\ Hz.}$$

Problem 5. *A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v_s ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, what is the maximum value of v_s upto which he can hear the whistle.* **[AIEEE 06]**

Solution. Here source moves towards the stationary observer.

The apparent frequency is

$$\nu' = \frac{v}{v - v_s} \times \nu$$

$$10{,}000 = \frac{300}{300 - v_s} \times 9500$$

or $\quad 300 - v_s = 285$

or $\quad v_s = 300 - 285 = \mathbf{15\ ms^{-1}}.$

Problem 6. *Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the Fig. 15.33. The speed of each pulse is 2 cm/s. After 2 seconds, what will be the nature of the total energy of the pulses?* **[IIT Screening 01]**

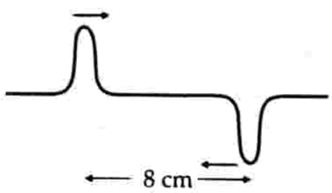


Fig. 15.33

Solution. After 2 seconds, the two pulses will be at the same location. They add up destructively, giving zero displacement at each point. The string becomes straight and so does not have any potential energy. Its total energy must be kinetic.

Problem 7. *Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. What is the ratio of the speed of sound in gas 1 to that in gas 2?* **[IIT Screening 2K]**

Solution. The speed of sound in a gas of molecular mass M is

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad i.e., \qquad v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \qquad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}.$$

Problem 8. *Two vibrating strings of the same material but lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency ν_1 and the other with frequency ν_2. What is the ratio ν_1/ν_2?* **[IIT Screening 2K]**

Solution. The frequency of vibration of the fundamental mode,

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

As T and l are the same for both wires, so

$$\nu \propto \frac{1}{lr}$$

$$\therefore \qquad \frac{\nu_1}{\nu_2} = \frac{l_2 r_2}{l_1 r_1} = \frac{2L \cdot r}{L \cdot 2r} = \mathbf{1}.$$

Problem 9. *The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1, and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then how are E_1 and E_2 related?* **[IIT Screening 2K]**

Solution. The equation of a stationary wave is

$$y = A \sin(kx) \sin(\omega t)$$

Here, for y_1, $k_1 = \pi/L$, $\omega_1 = \omega$

$$\therefore \qquad v_1 = \frac{\omega_1}{k_1} = \frac{\omega L}{\pi}.$$

For y_2, $k_2 = 2\pi/L$, $\omega_2 = 2\omega$

$$\therefore \qquad v_2 = \frac{\omega_2}{k_2} = \frac{\omega L}{\pi} = v_1.$$

Thus the wave velocities are the same in both cases. Also, they have the same amplitude. The frequency for y_2 is twice the frequency for y_1.

As energy $\propto$ (frequency)^2

$$\therefore \qquad E_2 = \mathbf{4E_1}.$$

Problem 10. *A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges, find the value of M.* **[IIT Screening 02]**

Solution. $\nu_p = \dfrac{p}{2l}\sqrt{\dfrac{T}{m}}$

In the first case, $\nu_5 = \dfrac{5}{2l}\sqrt{\dfrac{9g}{m}}$

In the second case, $\nu_3 = \dfrac{3}{2l}\sqrt{\dfrac{Mg}{m}}$

As the two modes are in resonance with the same tunning fork, so

$$\frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}}$$

$$15 = 3\sqrt{M} \qquad \text{or} \qquad M = \mathbf{25\ kg}.$$

Problem 11. *In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.* **[IIT Screening 03]**

Solution. End correction

$$= \frac{L_2 - 3L_1}{2} = \frac{0.35 - 3 \times 0.1}{2} = 0.025 \text{ m} = \mathbf{2.5\ cm.}$$

Problem 12. *A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then find the ratio f_1/f_2.* **[IIT Screening 2K ; Central Schools 11]**

Solution. For the stationary observer,

$$f' = \frac{v}{v - v_s} \times f$$

$$\therefore \qquad f_1 = \frac{340}{340 - 34} \times f$$

and $$f_2 = \frac{340}{340 - 17} \times f$$

Hence $$\frac{f_1}{f_2} = \frac{340 - 17}{340 - 34} = \mathbf{\frac{19}{18}}.$$

Problem 13. *A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz while approaching the same siren. What is the ratio of the velocity of train B to that train A?* **[IIT Screening 02]**

Solution. When the observer approaches the stationary source,

$$\nu' = \frac{v + v_0}{v}\nu$$

For train A, $5.5 = \frac{v + v_A}{v} \times 5$ or $v_A = 0.1v$

For train B, $6.0 = \frac{v + v_B}{v} \times 5$ or $v_B = 0.2v$

$\therefore \quad \frac{v_B}{v_A} = 2.$

Problem 14. *A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.* **[IIT Screening 03]**

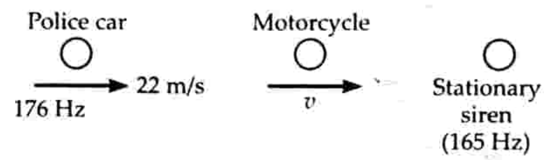


Fig. 15.34

Solution. Frequency of the police horn heard by the motorcyclist,

$$\nu_1 = \frac{330 - v}{330 - 22} \times 176$$

Frequency of the stationary siren heard by the motorcyclist,

$$\nu_2 = \frac{330 + v}{330} \times 165$$

For no beats,

$$\nu_1 = \nu_2$$

$$\frac{330 - v}{330 - 22} \times 176 = \frac{330 + v}{330} \times 165$$

or $\quad v = \mathbf{22\ ms^{-1}}.$

Problem 15. *A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air.* **[IIT Mains 03]**

Solution. In the fundamental mode of vibration,

$$(l + 0.3\text{ D}) = \frac{\lambda}{4} = \frac{v}{4\nu}$$

$$\therefore \quad v = 4\nu\,(l + 0.3\text{ D})$$

$$= 4 \times 480\,(0.16 + 0.3 \times 0.05)\ \text{ms}^{-1}$$

$$= \mathbf{336\ ms^{-1}}.$$

Problem 16. *The length of a sonometer wire is 0.75 m and density 9×10^3 kg m^{-3}. It can bear a stress of 8.1×10^8-N/m^2 without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire?* **[IIT 90]**

Solution. Here $L = 0.75$ m, $\rho = 9 \times 10^3$ kg m^{-3}, Stress $= 8.1 \times 10^9$ Nm^{-2}

Let a be the area of cross-section of the wire, then Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{\text{Stress} \times a}{a \times 1 \times \rho}}$$

$$= \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\rho}} = \frac{1}{2 \times 0.75}\sqrt{\frac{8.1 \times 10^8}{9 \times 10^3}} = \mathbf{200\ Hz.}$$

Problem 17. *A transverse sinusoidal wave of amplitude A, wavelength λ and frequency ν is travelling on a stretched string. The maximum speed of any point on the string is $\frac{v}{10}$, where v is the speed of propagation of the wave. If $A = 10^{-3}$ m and $v = 10$ ms^{-1}, then find the values of ν and λ.* **[IIT 98]**

Solution. Here $v_{max} = \frac{v}{10} = \frac{10}{10} = 1\ \text{ms}^{-1}$,

$$A = 10^{-3}\ \text{m}$$

Velocity amplitude or maximum velocity is given by

$$v_{max} = \omega A = 2\pi \nu A$$

$$\nu = \frac{v_{max}}{2\pi A} = \frac{1}{2\pi \times 10^{-3}} = \mathbf{\frac{10^3}{2\pi}\ Hz.}$$

Wavelength,

$$\lambda = \frac{v}{\nu} = \frac{10 \times 2\pi}{10^3} = \mathbf{2\pi \times 10^{-2}\ m.}$$

Problem 18. *An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of the open pipe?* **[IIT 96]**

Solution. Fundamental frequency of open pipe,

$$\nu_o = \frac{v}{2L}$$

Frequency of third harmonic of closed pipe,

$$\nu_c = \frac{3v}{4L}$$

$$\therefore \quad \frac{\nu_c}{\nu_o} = \frac{3}{2} \quad \text{or} \quad \nu_c = \frac{3}{2}\nu_o$$

Given $\nu_c = \nu_o + 100$

$$\therefore \quad \frac{3}{2}\nu_o = \nu_o + 100 \quad \text{or} \quad \nu_o = \mathbf{200\ Hz.}$$

Problem 19. *The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound in air = 330 ms^{-1}.* **[IIT 97]**

Solution. Let lengths of the open and closed organ pipes be L_0 and L_c respectively.

Fundamental frequency of closed pipe,

$$\nu_c = \frac{v}{4L_c} \qquad \therefore \qquad L_c = \frac{v}{4\nu_c}$$

But $\qquad v = 330\ \text{ms}^{-1}, \qquad \nu_c = 110\ \text{Hz}$

$$\therefore \qquad L_c = \frac{330}{4 \times 110} = 0.75\ \text{m} = \mathbf{75\ cm.}$$

Frequency of first overtone of open organ pipe,

$$\nu_0 = 2 \times \frac{v}{2L_0} = \frac{v}{L_0}$$

Frequency of first overtone of closed pipe,

$$\nu_c = 3 \times \frac{v}{4L_c} = 3 \times 110 = 330\ \text{Hz.}$$

But $\quad \nu_0 - \nu_c = 2.2\ \text{Hz}$

$$\therefore \qquad \frac{v}{L_0} - 330 = 2.2$$

or $$\frac{v}{L_0} = 332.2$$

or $$L_0 = \frac{v}{332.2} = \frac{330}{332.2} = 0.99\ \text{m} = \mathbf{99\ cm.}$$

Problem 20. *A metallic rod of length 1 m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the midpoint and those of the constituent waves in the rod.*

(Young's modulus $= 2 \times 10^{11}\ Nm^{-2}$, density $= 8000\ kg\ m^{-3}$). **[IIT 94]**

Solution. Velocity of longitudinal waves in the rod is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8000}} = 5000\ \text{ms}^{-1}$$

As there are two nodes on either side of the midpoint, so

$$L = \frac{5\lambda}{2} = 1\ \text{m}$$

or $$\lambda = 0.4\ \text{m}$$

Frequency,

$$\nu = \frac{v}{\lambda} = \frac{5000}{0.4} = 12500\ \text{Hz}$$

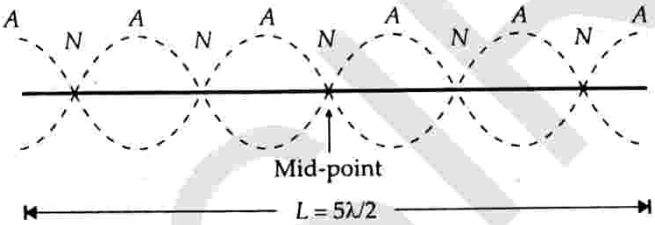


Fig. 15.35

$\because$ Amplitude of an antinode $= 2 \times$ Amplitude of a constituent wave

$$\therefore \qquad 2A = 2 \times 10^{-6}\ \text{m} \quad \text{or} \quad A = 1 \times 10^{-6}\ \text{m}$$

Equation for a stationary wave is

$$y = 2A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

But $\quad A = 1 \times 10^{-6}\ \text{m}, \quad x = 2\ \text{cm} = 0.02\ \text{m},$

$\lambda = 0.4\ \text{m}, \qquad 1/T = 12500\ \text{Hz}$

$$\therefore \qquad \mathbf{y = 2 \times 10^{-6} \cos \frac{\pi}{10} \sin 25000\,\pi t.}$$

This is the equation of motion at a point 2 cm from the mid-point.

The waves constituting the stationary waves are given by

$$y_{1,2} = A \sin 2\pi \left(\frac{t}{T} \mp \frac{x}{\lambda} \right)$$

or $$y_{1,2} = 1 \times 10^{-6} \sin 2\pi (12500\,t \mp 2.5\,x).$$

Guidelines to NCERT Exercises

15.1. *A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end ?*

Ans. Given $m = \frac{2.50}{20.0}\ \text{kg m}^{-1}$,

$T = 200\ \text{N}$

Speed of the transverse jerk is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200 \times 20.0}{2.50}} = \sqrt{1600} = 40\ \text{ms}^{-1}.$$

$\therefore$ Time taken by the jerk to reach the other end

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{20}{40} = \mathbf{0.5\ s.}$$

15.2. *A stone dropped from the top of a tower 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top ? Speed of sound in air = 340 ms^{-1} ; $g = 9.8\ ms^{-2}$.*

Ans. Let t be the time taken by the stone to reach the water surface.

Here $s = 300$ m, $u = 0$, $a = g = 9.8\ \text{ms}^{-2}$

As $s = ut + \frac{1}{2}at^2$

$\therefore \quad 300 = \frac{1}{2} \times 9.8 \times t^2$

or $\quad t^2 = \frac{2 \times 300}{9.8} = 61.2$

$\therefore \quad t = \sqrt{61.2} = 7.82$ s.

Time taken by the splash to reach from water surface to the top,

$$t' = \frac{\text{Distance}}{\text{Speed}} = \frac{300}{340} = 0.88\ \text{s}$$

$\therefore$ Total time taken by the splash to be heard at the top

$= t + t' = 7.82 + 0.88 =$ **8.7 s.**

15.3. *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20° C which is 343 ms^{-1}?*

Ans. Speed of a transverse wave in the steel wire is given by

$$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

or $\quad T = v^2$ m

Given $\quad v = 343\ \text{ms}^{-1}, \quad m = \frac{2.10}{12.0}\ \text{kg m}^{-1}$

$\therefore \quad T = (343)^2 \times \frac{2.10}{12.0} = \mathbf{2.06 \times 10^4\ N.}$

15.4. *Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air*

(*a*) *is independent of pressure,*

(*b*) *increases with temperature,*

(*c*) *increases with humidity.*

Ans. Refer answer to Q. 12 on page 15.11.

15.5. *You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - v\,t$ or $x + v\,t$, i.e., $y = F(x \pm v\,t)$. Is the converse true ? Examine if the following functions for y can possibly represent a travelling wave :*

(*i*) $(x - vt)^2$ (*ii*) $\log[(x + v\,t)/x_0]$

(*iii*) $\exp[-(x + v\,t)/x_0]$ (*iv*) $1/(x + v\,t)$.

Ans. If $y = f(x \pm vt)$ represents a travelling wave, then the converse may not be true *i.e.*, every function of $x - vt$ or $x + vt$ may not always represent a travelling wave. The basic requirement for a function to represent a travelling wave is that it must be finite for all values of x and t.

The functions (*i*), (*ii*) and (*iv*) are not finite for all values of x and t, hence they cannot represent a travelling wave. Only function (*iii*) satisfies the condition to represent a travelling wave.

15.6. *A bat emits ultrasonic sound of frequency 100 kHz in air. If this sound meets a water surface, what is the wavelength of (i) the reflected sound, (ii) the transmitted sound ? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.*

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Ans. Here $\nu = 100\ \text{kHz} = 10^5$ Hz,

$$v_a = 340\ \text{ms}^{-1}, \quad v_w = 1486\ \text{ms}^{-1}$$

Frequency of both the reflected and transmitted sound remains unchanged.

(*i*) Wavelength of reflected sound,

$$\lambda_a = \frac{v_a}{\nu} = \frac{340}{10^5} = \mathbf{3.4 \times 10^{-3}\ m.}$$

(*ii*) Wavelength of transmitted sound,

$$\lambda_w = \frac{v_w}{\nu} = \frac{1486}{10^5} = \mathbf{1.49 \times 10^{-2}\ m.}$$

15.7. *A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 kms^{-1} ? The operating frequency of the scanner is 4.2 MHz.*

Ans. Here $v = 1.7\ \text{kms}^{-1} = 1.7 \times 10^3\ \text{ms}^{-1}$,

$$\nu = 4.2\ \text{MHz} = 4.2 \times 10^6\ \text{Hz}$$

Wavelength,

$$\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = \mathbf{4.047 \times 10^{-4}\ m.}$$

15.8. *A transverse harmonic wave on a string is described by*

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4),$$

where x, y are in cm and t in s. The positive direction of x is from left to right.

(*i*) *Is this a travelling or a stationary wave ? If it is travelling, what are the speed and direction of its propagation ?*

(*ii*) *What are its amplitude and frequency ?*

(*iii*) *What is the initial phase at the origin ?*

(*iv*) *What is the least distance between two successive crests in the wave ?*

Ans. Given :

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4) \quad \text{...(1)}$$

The standard equation for a harmonic wave is

$$y(x, t) = A \sin\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x + \phi_0\right) \quad \text{...(2)}$$

Comparing equations (1) and (2), we get

$$A = 3.0, \quad \frac{2\pi}{T} = 36, \quad \frac{2\pi}{\lambda} = 0.018, \quad \phi_0 = \frac{\pi}{4}$$

(*i*) The given equation represents travelling wave propagating from *right to left* (as x term is +ve).

Speed of the wave,

$$v = \frac{\lambda}{T} = \frac{\lambda / 2\pi}{T / 2\pi} = \frac{1/0.018}{1/36}$$

$$= \frac{36}{0.018} = 2000 \text{ cm s}^{-1} = \mathbf{20\ ms^{-1}}.$$

(*ii*) Amplitude, $A = 3.0$ cm

Frequency, $\nu = \frac{1}{T} = \frac{36}{2\pi} = \frac{18}{3.14} = \mathbf{5.73\ s^{-1}}$.

(*iii*) Initial phase at the origin, $\phi = \frac{\pi}{4}$ **rad.**

(*iv*) Least distance between two successive crests is equal to wavelength.

$$\therefore \quad \lambda = \frac{2\pi}{0.018} = 349.0 \text{ cm} = \mathbf{3.49\ m.}$$

15.9. *For the wave described in Exercise 15.8 displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shapes of these graphs ? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase ?*

Ans. The transverse harmonic wave on a string is given by

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$$

Displacement-time graph for $x = 0$:

For $x = 0$, we have

$$y(0, t) = 3.0 \sin(36t + \pi/4)$$

$$a = 3 \text{ cm}, \ \phi_0 = \pi/4 \text{ and } \omega = 36 \text{ rad s}^{-1}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{36} = \frac{\pi}{18} \text{ s.}$$

The displacement at different instants of time will be as follows :

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$-\frac{3}{\sqrt{2}}$	-3	$-\frac{3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

The y-t graph will be as shown in Fig. 15.36.

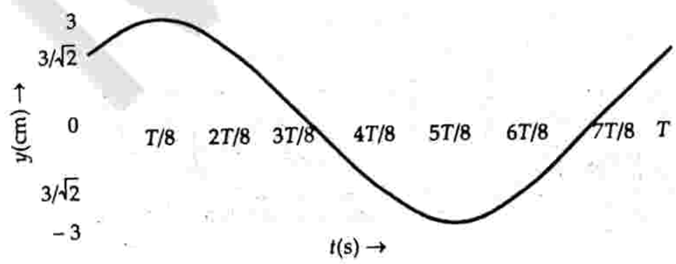

Fig. 15.36

Thus, the y-t graph is sinusoidal in nature.

Similarly, we can obtain y-t graphs for $x = 2$ cm and $x = 4$ cm. It is seen that all the three graphs are sinusoidal. They have same amplitude and frequency, but they differ in initial phases.

15.10. *For a travelling harmonic wave*

$$y = 2.0 \cos(10t - 0.0080x + 0.35)$$

where x and y are in cm and t in s. What is the phase difference between oscillatory motion at two points separated by a distance of (i) 4 m, (ii) 0.5 m, (iii) $\frac{\lambda}{2}$, *(iv)* $\frac{3\lambda}{4}$ *?*

Ans. Given $y = 2.0 \cos(10t - 0.0080x + 0.35)$ ...(1)

The standard equation of travelling harmonic wave is

$$y = A \cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right] \quad ...(2)$$

Comparing equations (1) and (2), we get

$$\frac{2\pi}{\lambda} = 0.0080 \text{ or } \lambda = \frac{2\pi}{0.0080} \text{ cm}$$

$$= \frac{2\pi}{0.0080 \times 100} \text{ m} = \frac{2\pi}{0.80} \text{ m}$$

Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda} \times \Delta x$$

(*i*) When $\Delta x = 4$ m

$$\therefore \quad \Delta\phi = \frac{2\pi}{2\pi} \times 0.80 \times 4 = \mathbf{3.2\ rad.}$$

(*ii*) When $\Delta x = 0.5$ m

$$\therefore \quad \Delta\phi = \frac{2\pi}{2\pi} \times 0.80 \times 0.5 = \mathbf{0.40\ rad.}$$

(*iii*) When $\Delta x = \frac{\lambda}{2}$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \textbf{ rad.}$$

(*iv*) When $\Delta x = \frac{3\lambda}{4}$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \textbf{ rad.}$$

15.11. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos(120\pi t)$$

where x, y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} *kg. Answer the following :*

(*a*) *Does the function represent a travelling or a stationary wave ?*

(*b*) *Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave ?*

(*c*) *Determine the tension in the string.*

Ans. The equation for transverse displacement is

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t \quad ...(i)$$

(*a*) The equation (*i*) represents a stationary wave as it involves the product of two separate harmonic functions of *x* and *t*.

(*b*) Stationary waves are formed by the superposition of two waves of same frequency travelling in opposite directions. Let the two waves be represented as

$$y_1 = A \sin \frac{2\pi}{\lambda}(vt - x)$$

and $$y_2 = -A \sin \frac{2\pi}{\lambda}(vt + x)$$

The resultant stationary wave is given by

$$y = y_1 + y_2$$

$$= A\left[\sin \frac{2\pi}{\lambda}(vt - x) - \sin \frac{2\pi}{\lambda}(vt + x)\right]$$

$$= -2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad ...(ii)$$

Comparing the equations (*i*) and (*ii*), we get

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \quad \text{or} \quad \lambda = 3 \text{ m}$$

and $$\frac{2\pi}{\lambda} v = 120\pi$$

or $$v = 60\lambda = 60 \times 3 = \mathbf{180 \ ms^{-1}}.$$

Frequency,

$$\nu = \frac{v}{\lambda} = \frac{180}{3} = \mathbf{60 \ Hz.}$$

(*c*) Velocity of transverse wave in a string is given by

$$v = \sqrt{\frac{T}{m}}$$

But $$m = \frac{3.0 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg m}^{-1},$$

$$v = 180 \text{ ms}^{-1}$$

$$\therefore \quad T = v^2 m = (180)^2 \times 2 \times 10^{-2} = \mathbf{648 \ N.}$$

15.12. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t,$$

where x, y are in m and t is in s.

(*i*) *Do all the points on the string oscillate with the same* (*a*) *frequency,* (*b*) *phase,* (*c*) *amplitude ? Explain your answers.*

(*ii*) *What is the amplitude of a point 0.375 m away from one end ?*

Ans. (*i*) The transverse displacement is given by

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t$$

(*a*) The time dependent harmonic function $\cos 120\pi t$ of the stationary wave represents its frequency. As this function does not depend on *x*, so frequency of oscillation of all points on the string is same.

(*b*) The phase of all the points on the string is same for the reasons similar to (*a*).

(*c*) The amplitude of stationary wave is given by

$$A = 0.06 \sin \frac{2\pi}{3} x \quad \text{[time independent part]}$$

As *A* depends on *x*, amplitude of all the points on the string is *not same.*

(*ii*) Now, amplitude at a point 0.375 m away from one end is given by

$$A = 0.06 \sin \frac{2\pi}{3} \times 0.375 = 0.06 \sin 0.7854$$

$$= 0.06 \times 0.707 = \mathbf{0.042 \ m.}$$

15.13. *Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all :*

(*a*) $y = 2\cos(3x)\sin(10t)$

(*b*) $y = 2\sqrt{x - vt}$

(*c*) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(*d*) $y = \cos x \sin t + \cos 2x \sin 2t.$

Ans. (*a*) As the function is the product of two separate harmonic functions of *x* and *t*, so it represents a stationary wave.

(*b*) It cannot represent any type of wave.

(*c*) Here $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

$$= 3\sin\theta + 4\cos\theta \quad [\theta = 5x - 0.5t]$$

If we put $A\cos\alpha = 3$ and $A\sin\alpha = 4$, then

$$y = a\sin(\theta + \alpha)$$

It represents a simple harmonic travelling wave of amplitude,

$$A = \sqrt{3^2 + 4^2} = 5 \quad \text{and} \quad \alpha = \tan^{-1}(4/3).$$

(*d*) It represents the superposition of two stationary waves, one represented by $\cos x \sin t$ and another by $\cos 2x \sin 2t$.

15.14. *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-1}. What is*

(*i*) *the speed of a transverse wave on the string, and*

(*ii*) *the tension in the string ?*

Ans. Length of the wire,

$$L = \frac{\text{Mass of the wire (M)}}{\text{Linear density (m)}} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875 \text{ m}$$

Given : $\nu = 45$ Hz, $L = 0.875$ m,

$$m = 4.0 \times 10^{-2} \text{ kg m}^{-1}$$

(*i*) As $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$

$\therefore \quad 45 = \frac{1}{2 \times 0.875}\sqrt{\frac{T}{4.0 \times 10^{-2}}}$

Squaring,

$$(45)^2 = \frac{1}{(2 \times 0.875)^2} \times \frac{T}{4.0 \times 10^{-2}}$$

or $T = (45)^2 \times (2 \times 0.875)^2 \times 4.0 \times 10^{-2}$

$= \mathbf{248\ N}.$

(*ii*) Speed of the transverse wave,

$$v = 2\nu L = 2 \times 45 \times 0.875 = \mathbf{78.75\ ms^{-1}}.$$

15.15. *A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz), when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.*

Ans. The frequency of nth mode of vibration for a closed end pipe is given by

$$\nu_n = \frac{(2n-1)\,v}{4L}, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

Suppose the lengths $L_1 = 25.5$ cm and $L_2 = 79.3$ cm of the resonance columns correspond to $n = n_1$ and $n = n_2$ respectively. Then

$$\nu = \frac{(2n_1 - 1)\,v}{4L_1} = \frac{(2n_2 - v)}{4L_2} \quad \ldots(1)$$

or $340 = \frac{(2n_1 - 1)v}{4 \times 25.5} = \frac{(2n_2 - 1)\,v}{4 \times 79.3}$

or $\frac{2n_1 - 1}{2n_2 - 1} = \frac{25.5}{79.3} \simeq \frac{1}{3}$

This is possible if $n_1 = 1$ and $n_2 = 2$. Thus the resonance length 25.5 cm corresponds to the fundamental note and 79.3 cm corresponds to first overtone or third harmonic. From equation (1), we get

$$v = \frac{4L_1\,\nu}{2n_1 - 1} = \frac{4 \times 25.5 \times 340}{2 \times 1 - 1}$$

$$= 34680 \text{ cm s}^{-1} = \mathbf{346.8\ ms^{-1}}.$$

15.16. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel ?*

Ans. For the fundamental mode of vibration

$$\lambda = 2L = 2 \times 100 = 200 \text{ cm} = 2 \text{ m}$$

Frequency,

$$\nu = 2.53 \text{ kHz} = 2530 \text{ Hz}$$

$\therefore$ Speed of sound,

$$v = \nu\lambda = 2530 \times 2 = 5060 \text{ ms}^{-1}$$

$$= \mathbf{5.06\ km\ s^{-1}}$$

15.17. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will this same source be in resonance with the pipe if both ends are open ? (Speed of sound = 340 ms^{-1}).*

Ans. Length of the pipe $L = 20$ cm $= 0.20$ m,

Speed of sound $v = 340 \text{ ms}^{-1}$

Fundamental frequency of vibration of the closed organ pipe is

$$\nu = \frac{v}{4L} = \frac{340}{4 \times 0.20} = 425 \text{ Hz}$$

Hence the fundamental mode of the closed organ pipe may be reasonably excited by a source of frequency 430 Hz.

Fundamental frequency of vibration of the open organ pipe is

$$\nu' = \frac{v}{2L} = \frac{340}{2 \times 0.20} = 850 \text{ Hz}$$

Hence the same source of frequency 430 Hz will not be in resonance with open organ pipe.

15.18. *Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B ?*

Ans. Frequency of $A = 324$ Hz

Beat frequency = 6 Hz

$\therefore$ Possible frequencies of B

$= 324 \pm 6 = 330$ or 318 Hz

When the tension reduces, frequency of string A decreases ($\nu \propto \sqrt{T}$). If the original frequency of B is 318 Hz, the beat frequency should decrease on reducing the tension in A. This is given to be the case as beat frequency decreases from 6 Hz to 3 Hz.

$\therefore$ Frequency $B = \mathbf{318\ Hz}$.

15.19. *Explain why (or how) :*

(*a*) *in a sound wave, a displacement node is a pressure antinode and vice versa,*

(*b*) *bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",*

(*c*) *a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,*

(*d*) *solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and*

(*e*) *the shape of a pulse gets distorted during propagation in a dispersive medium.*

Ans. (*a*) At a displacement node (the point of zero displacement), the variation of pressure is maximum. Hence the displacement node is the pressure antinode and vice-versa.

(*b*) Bats can produce and detect ultrasonic waves (sound waves of frequencies above 20 kHz). (*i*) From the interval of time between their producing the waves and receiving the echo after reflection from an object, they can estimate the distance of the object from them. (*ii*) From the intensity of the echo, they can estimate the nature and size of the object. (*iii*) Also, from the small time interval between the reception of the echo by their two ears, they can determine the direction of the object.

(*c*) The instruments produce different overtones (integral multiples of fundamental frequency). Hence the *quality* of sound produced by the two instruments of even same fundamental frequency is different.

(*d*) Solids have both volume and shear elasticity. So both longitudinal and transverse waves can propagate through them. On the other hand, gases have only volume elasticity and not shear elasticity. So only longitudinal waves can propagate through them.

(*e*) When a pulse passes through a dispersive medium, the wavelength of the wave changes. Consequently, the shape of the pulse changes *i.e.*, it gets distorted.

15.20. *A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air.*

(*i*) *What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of* $10\ ms^{-1}$, *(b) recedes from the platform with a speed of* $10\ ms^{-1}$ *?*

(*ii*) *What is the speed of sound in each case ?*

(Speed of sound in still air = $340\ ms^{-1}$*).*

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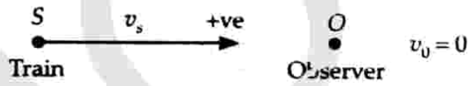
Ans. Here $\nu = 400$ Hz, $v_s = 10\ ms^{-1}$,

$$v_0 = 0, \quad v = 340\ ms^{-1}.$$

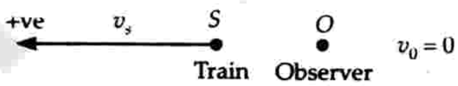
(*i*) (*a*) When the train approaches the stationary observer, $v_0 = 0$, $v_s = +10\ ms^{-1}$.



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 400$$

$$= \mathbf{412.12\ Hz.}$$

(*b*) When the train recedes from the observer, $v_0 = 0$, $v_s = -10\ ms^{-1}$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 10} \times 400$$

$$= \mathbf{388.57\ Hz.}$$

(*ii*) Speed of sound in each case = $\mathbf{340\ ms^{-1}}$.

15.21. *A train standing in a station yard blows a whistle of frequency 400 Hz in still air. (a) A wind starts blowing in the direction from the yard to the station with a speed of* $10\ ms^{-1}$. *What are the frequency, wavelength and speed of the sound for an observer standing on the station platform ? (b) Is the situation exactly equivalent to the case, when the air is still and the observer runs towards the yard at a speed of* $10\ ms^{-1}$ *? Take speed of sound in still air =* $340\ ms^{-1}$.

Ans. Here $\nu = 400$ Hz, $v = 340\ ms^{-1}$,

$$v_m = 10\ ms^{-1}$$

(*a*) As wind is blowing in the direction of sound, therefore, for an observer standing on the platform,

Speed of sound,

$$v' = v + v_m = 340 + 10 = \mathbf{350\ ms^{-1}}.$$

As there is no relative motion between the source and the observer, the frequency of sound remains unchanged.

Frequency of sound,

$$\nu = 400 \text{ Hz}$$

Wavelength of sound,

$$\lambda' = \frac{v'}{\nu} = \frac{350}{400} = \mathbf{0.875\ m.}$$

(*b*) When the observer moves towards the stationary engine (source) in still air,

$$v_0 = -10\ ms^{-1}, \quad v_s = 0$$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 10}{340 - 0} \times 400$$

$$= \frac{350}{340} \times 400 = \mathbf{411.8\ Hz.}$$

As wavelength of sound waves is not affected by motion of the observer, it remains unchanged.

Speed of sound relative to the observer,

$$v'' = 340 + 10 = \mathbf{350\ ms^{-1}}.$$

Thus the situations (*a*) and (*b*) are not equivalent.

15.22. *A travelling harmonic wave on a string is described by*

$$y = 7.5 \sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

(*i*) *What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s ? Is this velocity equal to the velocity of wave propagation ?*

(*ii*) *Locate the points of the string, which have the same transverse displacement and velocity as the x = 1 cm point at t = 2 s, 5 s, 11 s.*

Ans. Given $y = 7.5 \sin(0.0050x + 12t + \pi/4)$ ...(1)

$$= 7.5 \sin\left[0.0050\left\{\frac{12}{0.0050}t + x\right\} + \pi/4\right]$$

The standard equation of a travelling wave is

$$y = A \sin\left[\frac{2\pi}{\lambda}(vt + x) + \pi/4\right] \quad ...(2)$$

Comparing equations (1) and (2), we get

$$A = 7.5\,\text{cm}, \quad v = \frac{12}{0.0050}\,\text{cm s}^{-1}$$

and $\frac{2\pi}{\lambda} = 0.0050\,\text{cm}^{-1}$

(*i*) At $x = 1\,\text{cm}$ and $t = 1\,\text{s}$, displacement is

$$y = 7.5\sin(0.0050 \times 1 + 12 \times 1 + \pi/4)$$
$$= 7.5\sin 12.79$$
$$= 7.5 \times 0.2222 = \mathbf{1.67\ cm.}$$

Velocity of oscillation of the particle is

$$u = \frac{dy}{dt} = \frac{d}{dt}[7.5\sin(0.0050x + 12t + \pi/4)]$$
$$= 7.5 \times 12\cos(0.0050x + 12t + \pi/4)$$
$$= 90\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At $x = 1\,\text{cm}$ and $t = 1\,\text{s}$,

$$u = 90\cos(0.0050 \times 1 + 12 \times 1 + \pi/4)$$
$$= 90\cos 12.79$$
$$= 90 \times 0.9751 = \mathbf{87.76\ cm\ s^{-1}}.$$

Velocity of wave propagation is

$$v = \frac{12}{0.0050} = \mathbf{2400\ cm\ s^{-1}}$$

∴ Velocity of oscillation of a point is not equal to the velocity of wave.

(*ii*) As $\frac{2\pi}{\lambda} = 0.0050$

$$\therefore \quad \lambda = \frac{2\pi}{0.0050} = \mathbf{1256.64\ cm}$$

All points located at distance $n\lambda$ (where n is an integer) from the point $x = 1\,\text{cm}$ have the same transverse displacement and velocity.

15.23. *A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite : (i) frequency, (ii) wavelength, (iii) speed of propagation ? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz ?*

Ans. (*a*) A narrow sound pulse such as a short *pip* by a whistle does not have a definite wavelength or frequency. But being a sound wave, it has a definite speed (in a non-dispersive medium).

(*b*) The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetition.

15.24. *One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.*

Ans. Tension in the string,

$$T = 90 \times 9.8 = 882\ \text{N}$$

Mass per unit length of the string,

$$m = 8.0 \times 10^{-3}\ \text{kg m}^{-1}.$$

Frequency of the wave,

$$\nu = 250\ \text{Hz}$$

Amplitude of the wave,

$$A = 5.0\,\text{cm} = 0.05\ \text{m}$$

The velocity of the transverse wave along the string is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{882}{8 \times 10^{-3}}} = 3.32 \times 10^2\ \text{ms}^{-1}$$

Angular frequency,

$$\omega = 2\pi\nu = 2 \times 3.142 \times 256$$
$$= 16.1 \times 10^2\ \text{rad s}^{-1}$$

As $\quad v = \frac{\omega}{k}$

$$\therefore \quad k = \frac{\omega}{v} = \frac{16.1 \times 10^2}{3.32 \times 10^2} = 4.84\ \text{rad m}^{-1}$$

As the wave propagates along positive X-axis, so the displacement equation is

$$y = A\sin(\omega t - kx)$$

or $\quad y = 0.05\sin(16.1 \times 10^2 t - 4.84x)$, x and y are in m.

15.25. *A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 kmh^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 ms^{-1}.*

Ans. Frequency, $\nu = 40$ kHz,

Speed of sound = 1450 ms^{-1}

Speed of enemy submarine = 360 km h^{-1} = 100 ms^{-1}

Firstly sound is observed by enemy submarine. Here observer (enemy submarine) is moving towards the source (SONAR), so $v_0 = -100\ \text{ms}^{-1}$, $v_s = 0$. Frequency of sonar waves received by the enemy submarine,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{1450 + 100}{1450 - 0} \times 40$$
$$= \frac{1550}{1450} \times 40 = 42.76\ \text{kHz}$$

After the sound is reflected, enemy submarine acts as a source of sound of frequency ν'. This source moves with a speed of 100 ms^{-1} towards the observer (SONAR), so $v_s = +100\ \text{ms}^{-1}$, $v_0 = 0$. Frequency of sound reflected by the enemy submarine,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu' = \frac{1450 - 0}{1450 - 100} \times 42.76$$
$$= \mathbf{45.93\ kHz.}$$

15.26. *Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, how far away does the earthquake occur?*

Ans. Suppose the earthquake occurs at a distance of x km from the seismograph.

Speed of S wave = 4.0 km s^{-1}

Time taken by the S wave to reach the seismograph

$$= \frac{x}{4} \text{ s}$$

Speed of P wave = 8.0 km s^{-1}

Time taken by the P wave to reach the seismograph

$$= \frac{x}{8} \text{ s}$$

But $\frac{x}{4} - \frac{x}{8} = 4 \times 60 \text{ s}$

$\therefore$ x = **1920 km.**

15.27. *A bat is flitting about in a cave, navigating via ultrasonic bleeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?*

Ans. As the bat approaches the stationary flat wall surface, the apparent frequency is

$$\nu' = \frac{v}{v - v_s} \nu$$

The stationary flat surface (source) reflects the sound of frequency ν' to the bat (observer) moving towards the flat surface. So the apparent frequency is

$$\nu'' = \frac{v + v_0}{v} \times \nu' = \frac{v + v_0}{v} \times \frac{v}{v - v_s} \times \nu = \frac{v + v_0}{v - v_s} \times \nu$$

$$= \frac{v + 0.03v}{v - 0.03v} \times \nu$$

$$= \frac{1.03}{0.97} \times 40 \text{ kHz}$$

$$= \mathbf{42.47 \text{ kHz}}$$

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Define the term wave motion.
2. What are mechanical waves ?
3. What do mechanical waves transfer : energy, matter, information or none ?
4. Which type of waves do not require a material medium for their propagation ?
5. Give some common examples of electromagnetic waves.
6. What is the speed of electromagnetic waves in vacuum ?
7. What are matter waves ?
8. What is the angle between particle velocity and wave velocity in (*i*) transverse wave (*ii*) longitudinal wave ?
9. Which type of elasticity a medium must possess for transverse wave motion to be possible ?
10. Amongst solids, liquids and gases, in which type of media, transverse wave motion is possible ?
11. Amongst solids, liquids and gases, in which type of media, longitudinal wave motion can be transmitted ? **[Delhi 05]**
12. What is the nature of the sound waves ?
13. Among reflection, refraction, diffraction, interference and polarization, which is the sole characteristic of transverse waves ?
14. When the wire of a sonometer is plucked, what is the nature of the waves in (*i*) the string (*ii*) in air ?
15. What is the minimum distance between two points in a wave having a phase difference 2π ? **[Delhi 06]**
16. What is the phase difference between two nearest crests (or troughs) ?
17. A harmonic wave travelling in a medium has a period T and wave-length λ. How are λ and T related ?
18. A harmonic wave travelling in a medium has a period T and wave-length λ. How far does the wave travel in time T ?
19. What is the relation between frequency and wave-length ?
20. The time period of a vibrating source producing sound is 0.01 s. If the velocity of sound is 340 ms^{-1}, calculate the wave-length.

21. The density of oxygen is 16 times the density of hydrogen. What is the relation between the speeds of sound in two gases ? **[Chandigarh 08]**
22. What is the audible range of frequency ?
23. What is the wavelength range of audible sound and visible light ? **[Delhi 05]**
24. What is the effect of pressure on the speed of sound in air ?
25. What is the increase in the speed of sound in air when the temperature of air rises by 1°C ?
26. Can mechanical waves travel through vacuum ?
27. Draw a graph between the pressure of a gas and the speed of sound waves passing through that gas.
28. Does a vibrating body always produce sound ?
29. How far the consecutive nodes are separated from each other ?
30. What is the distance between a node and the nearest antinode ?
31. What is the phase difference between particles being on either side of a node ?
32. What will be the effect on the frequency of a sonometer wire if the tension is decreased by 2% ?
33. What is the minimum frequency with which a string of length L stretched under tension T can vibrate ?
34. Fundamental frequency of oscillation of a close pipe is 400 Hz. What will be the fundamental frequency of oscillation of an open pipe of same length ? **[Delhi 04]**
35. The fundamental frequency of an open organ pipe is 512 Hz. What will be its fundamental frequency if its one end is closed ?
36. The frequency of the fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths ?
37. The frequency of the first overtone of a closed organ pipe is same as that of the first overtone of an open organ pipe. What is the ratio between their lengths ?
38. An organ pipe produces a fundamental frequency of 128 Hz. When blown forcefully, it produces first overtone of 384 Hz. Is the pipe open or closed ?
39. How will the fundamental frequency of a closed organ pipe be affected if instead of air it is filled with a gas heavier than air ?
40. In an open organ pipe, third harmonic is 450 Hz. What is frequency of fifth harmonic ? **[Delhi 11]**
41. What is the main difference between a flute and a violin ?
42. Where to pluck and where to touch a stretched string to excite its first overtone ?
43. What is beat frequency ?
44. What is the essential condition for the formation of beats ?
45. Two sound sources produce 12 beats in 4 seconds. By how much do their frequencies differ ? **[Delhi 99 ; Chandigarh 08]**
46. What is the exact speed of light in vacuum ?
47. Give one use of beat phenomenon. **[Central Schools 03]**
48. State the principle of superposition of waves. **[Delhi 03C]**
49. State the factors on which the speed of a wave travelling along a stretched ideal string depends. **[Delhi 03]**
50. What are harmonics ?
51. What is Doppler effect ?
52. Velocity of sound in air at NPT is 332 m/s. What will be the velocity, when pressure is doubled and temp. is kept constant ? **[Central Schools 09]**
53. Name two instruments based on superposition of waves. **[Himachal 05C]**
54. Two sounds of very close frequencies, say 256 Hz and 260 Hz are produced simultaneously. What is the frequency of resultant sound and also write the number of beats heard in one second ? **[Central Schools 08]**
55. The frequencies of two tuning forks A and B are 250 Hz and 255 Hz respectively. Both are sounded together. How many beats will be heard in 5 seconds ? **[Delhi 12]**

Answers

1. Wave motion is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their mean position, the disturbance being handed over from one particle to the next.
2. The waves which require an elastic or material medium for their propagation are called mechanical waves.
3. Energy and information.
4. Electromagnetic waves.

5. The common examples of electromagnetic waves are visible and ultraviolet light, radio waves, microwaves, X-rays, infra-red light, etc.

6. All electromagnetic waves travel through vacuum at the same speed, $c = 29, 97, 92, 458\ ms^{-1}$.

7. Matter waves are the waves associated with moving electrons, protons, neutrons and other fundamental particles, and even atoms and molecules.

8. Angle between particle velocity and wave velocity for transverse wave is $\pi/2$. That in case of longitudinal wave is either zero or π.

9. Elasticity of shape.

10. Solids.

11. In all the three types of media *i.e.*, solids, liquids and gases.

12. Sound waves travel through any medium in the term of longitudinal waves.

13. Polarization.

14. The waves produced in the string are transverse and that in the air are longitudinal.

15. One wavelength (λ).

16. The phase difference between two nearest crests is 2π rad.

17. Wave velocity,
$$v = \frac{\lambda}{T} \quad \text{or} \quad \lambda = vT.$$

18. By definition, the wave will travel distance λ in time T (time period).

19. Wave velocity = Frequency × wavelength

i.e., $v = \nu\lambda$.

20. $\lambda = vT = 340 \times 0.01 = \mathbf{3.4\ m}$.

21. $$\frac{v_H}{v_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

or $v_H = 4v_O$.

22. The audible range of frequency is 20 Hz to 20 kHz.

23. (*i*) The wavelength range of audible sound in air is from 16.6×10^{-3} m to 16.6 m.

(*ii*) The wavelength range of visible light is from 4×10^{-7} m to 8×10^{-7} m.

24. The increase of pressure has no effect on the speed of sound in air.

25. The speed of sound increases by $0.61\ ms^{-1}$ for every 1°C rise in temperature of the air.

26. No, mechanical waves cannot travel through vacuum.

27. As shown in Fig. 15.37, the graph drawn between the pressure of a gas and the speed of sound

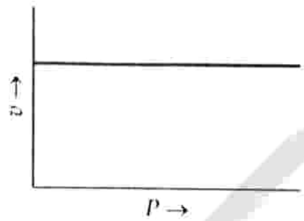


Fig. 15.37

through the gas is a straight line parallel to the pressure axis, because the speed of sound is independent of the pressure of the gas.

28. No. A vibrating body will produce sound only when its frequency of oscillation lies between 20 Hz to 20 kHz.

29. The separation between two consecutive nodes is $\lambda/2$.

30. The distance between a node and the nearest antinode is $\lambda/4$.

31. π rad.

32. As $\nu \propto \sqrt{T}$, so frequency decreases by 1% when tension is decreased by 2%.

33. Minimum frequency,
$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}},$$
where m is the mass per unit length of the string.

34. 800 Hz.

35. 256 Hz.

36. 1 : 2.

37. 3 : 4.

38. Closed pipe, because the first overtone is 3 times the fundamental frequency.

39. The fundamental frequency will decrease, because
$$\nu = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}}$$
i.e., $$\nu \propto \frac{1}{\sqrt{\rho}}.$$

40. Third harmonic = 3ν = 450 Hz

∴ Fundamental frequency, ν = 150 Hz

Fifth harmonic = 5ν = **750 Hz**

41. Flute is an organ pipe while violin is a musical instrument based on stretched string.

42. The wire should be plucked at its one-fourth length and touched at its centre.

43. The number of beats produced per second is called beat frequency.

44. The difference in frequency of the two sound waves should not exceed 10.

45. Number of beats produced per second,

$$b = \frac{12}{4} = 3\,s^{-1}.$$

$\therefore \quad \nu_1 - \nu_2 = b = 3.$

46. Speed of light in vacuum,

$$c = 29{,}97{,}92{,}458\,ms^{-1}.$$

47. The phenomenon of beats is used in tuning two musical instruments.

48. The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y_1 = y_1 + y_2 + y_3 + \ldots + y_n.$$

49. The speed of a wave travelling on a string depends on

(*i*) the tension (T) in string and

(*ii*) its mass per unit length (m)

$$v = \sqrt{\frac{T}{m}}.$$

50. Harmonics are the notes of frequencies which are integral multiples of the fundamental frequencies.

51. The apparent change in the frequency of sound when the source, observer and the medium are in relative motion, is called Doppler effect.

52. 332 m/s, because pressure has no effect on the velocity of sound in air.

53. (*i*) Sonometer

(*ii*) Organ pipe.

54. Frequency of resultant sound,

$$\nu_{av} = \frac{\nu_1 + \nu_2}{2}$$

$$= \frac{256 + 260}{2} = \mathbf{258\ Hz}.$$

Number of beats heard in one second

$$= 260 - 256 = \mathbf{4}.$$

55. Number of beats heard in 5 seconds

$$= (255 - 250) \times 5 = \mathbf{25}.$$

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the term wave motion. Give four important characteristics of wave motion. **[Chandigarh 03, 04]**

2. What are mechanical, electromagnetic and matter waves ? Give an example of each type.

3. What are transverse and longitudinal wave motions ? Give an example for each type.

4. Mention the important properties which a medium must possess for the propagation of mechanical waves through it.

5. Through what type of media, can (*i*) transverse waves and (*ii*) longitudinal waves be transmitted ? Explain.

6. Given below are some examples of wave motion. State in each case if the motion is transverse, longitudinal or a combination of both.

(*a*) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.

(*b*) Wave produced in a cylinder containing water by moving its piston back and forth.

(*c*) Wave produced by a motorboat sailing in water.

(*d*) Ultrasonic waves in air produced by a vibrating quartz crystal. **[Delhi 03]**

7. Derive a relation between wave velocity, frequency and wavelength.

8. On the basis of dimensional considerations, write the formula for the speed of transverse waves on a stretched string.

9. On the basis of dimensional considerations, write the formula for the speed of transverse waves in a solid.

10. Write Newton's formula for the speed of sound in air. What was wrong with this formula ? What correction was made by Laplace in this formula ?

[Chandigarh 03 ; Delhi 02, 03C, 06]

11. The speed of longitudinal waves 'v' in a given medium of density ρ is given by the formula

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Use this formula to explain why the speed of sound in air

(*a*) is independent of pressure,

(*b*) increases with temperature, and

(*c*) increases with humidity. **[Delhi 03]**

12. Discuss the effect of temperature on the velocity of sound in gases. **[Delhi 96 ; Himachal 05]**

13. Show that the speed of sound in air increases by $61\,cms^{-1}$ for every 1°C rise of temperature.

14. What is the effect of (*i*) frequency and (*ii*) amplitude, on the speed of sound in air ?

15. Why does sound travel faster in moist air than in dry air ?

16. What is a plane progressive harmonic wave ? Establish displacement relation for a harmonic wave travelling along the positive direction of *X*-axis.

17. What is wave motion ? Derive the equation of a harmonic wave. **[Delhi 98]**

18. What do you mean by phase of a wave ? Discuss the phase change with (*i*) time and (*ii*) position. Hence define time period and wavelength of a wave.

19. Considering suitable examples, discuss the phase change when a wave is reflected from (*i*) a rigid boundary and (*ii*) a free or open boundary.

20. State and illustrate the principle of superposition of waves.

21. What are stationary waves ? State the necessary condition for the formation of stationary waves.

22. Write any three characteristics of stationary waves. **[Central Schools 08]**

23. State the laws of vibrations of stretched strings.

24. Prove analytically that in the case of an open organ pipe of length *L*, the frequencies of vibrating air column are given by

$$\nu = n(v/2L),$$

where *n* is an integer.

25. Describe the various modes of vibration in case of a closed end organ pipe. **[Himachal 08C]**

26. Prove analytically that in the case of a closed organ pipe of length *L*, the frequencies of the vibrating air column are given by $\nu = (2n+1)(v/4L)$, where *n* is an integer.

27. Describe a simple experiment for showing the formation of beats.

28. Explain how the phenomenon of beats may be used for finding the unknown frequency of a tuning fork.

29. Explain Doppler effect in sound. Obtain an expression for the apparent frequency of sound when the source is moving towards the stationary observer with a uniform velocity. **[Chandigarh 03, 04 ; Himachal 04, 05 ; Central Schools 05]**

30. Derive an expression for the apparent frequency of the sound when the observer moves towards a stationary source of sound with a uniform velocity.

31. Bats have no eyes, still they travel during night. Explain, why ? **[Himachal 07C]**

32. Explain Doppler effect in sound. Obtain an expression for apparent frequency of sound when source and listener are approaching each other. **[Delhi 05 ; Central Schools 10]**

33. Distinguish between transverse and longitudinal waves. **[Himachal 05]**

34. What is the effect of pressure on the speed of sound ? Justify your answer. **[Delhi 05]**

35. What is the nature of sound waves in air ? How is the speed of sound waves in atmosphere affected by the (*i*) humidity (*ii*) temperature ? **[Delhi 04]**

36. Give any three differences between progressive waves and stationary waves. **[Himachal 09C ; Delhi 11]**

37. What is a progressive wave ? Derive an expression which represents a progessive wave. **[Himachal 2K, 05C]**

38. What is the difference between velocity of wave and velocity of a particle ? Obtain the relation for the velocity of wave in wave motion. **[Chandigarh 07]**

39. From the equation $y = r\sin\frac{2\pi}{\lambda}(vt - x)$, establish the relation between particle velocity, and wave velocity. **[Chandigarh 08]**

40. What are beats ? Prove that the number of beats produced per second by the two sound sources is equal to the difference between their frequencies. **[Central Schools 08, 09]**

41. Show that in open organ pipe, all harmonics are present. **[Central Schools 07]**

42. Diagrammatically show first two modes of vibrations in case of an open organ pipe and write the ratio of their frequencies. **[Central Schools 12]**

Answers

1. Refer answer to Q. 1 on page 15.1.
2. Refer answer to Q. 2 on page 15.2.
3. Refer answer to Q. 4 on page 15.3.
4. Refer answer to Q. 5 on page 15.3.

5. Refer answer to Q. 6 on page 15.3.
6. Refer to solution of Problem 3 on page 15.65.
7. Refer answer to Q. 8 on page 15.4.
8. Refer answer to Q. 9(*a*) on page 15.6.
9. Refer answer to Q. 9(*b*) on page 15.7.
10. Refer answer to Q. 11 on page 15.11.
11. Refer answer to Q. 12 on page 15.11.
12. Refer answer to Q. 12(*iv*) on page 15.11.
13. Refer answer to Q. 12(*iv*) on page 15.11.
14. Refer answer to Q. 12(*vi*) and (*vii*) on page 15.12.
15. Refer answer to Q. 12(*iii*) on page 15.11.
16. Refer answer to Q. 13 on page 15.15.
17. Refer answer to Q. 1 on page 15.1 and Q. 13 on page 15.15.
18. Refer answer to Q. 14 on page 15.16.
19. Refer answer to Q. 17 on page 15.22.
20. Refer answer to Q. 20 on page 15.23.
21. Refer answer to Q. 21 on page 15.24.
22. Refer answer to Q. 24 on page 15.26.
23. Refer answer to Q. 28 on page 15.30.
24. Refer answer to Q. 31 on page 15.36.
25. Refer answer to Q. 32 on page 15.36.
26. Refer answer to Q. 33 on page 15.37.
27. Refer answer to Q. 37 on page 15.42.
28. Refer answer to Q. 38(*i*) on page 15.42.
29. Refer answer to Q. 41 on page 15.49.
30. Refer answer to Q. 42 on page 15.50.
31. Refer answer to NCERT Exercise 15.19(*b*) on page 15.75.
32. Refer answer to Q. 42 on page 15.50.
33. Refer to solution of Problem 16 on page 15.67.
34. Refer answer to Q. 12(*i*) on page 15.11.
35. Refer answer to Q. 12(*iii*) and (*iv*) on page 15.11.
36. Refer to solution to Problem 20 on page 15.67.
37. Refer answer to Q.13 on page 15.15.
38. Refer answer to Q.15(*a*) on page 15.16.
39. Refer answer to Q.15(*a*) on page 15.16.
40. Refer answer to Q.34 and Q.36 on page 15.41.
41. Refer answer to Q.30 on page 15.35.
42. See Fig. 15.20 on page 15.35. Ratio $v_1 : v_2 = 1 : 2$.

Type C : Long Answer Questions **5 Marks Each**

1. On the basis of spring model, explain the propagation of a sound in (*i*) air and (*ii*) solids.
2. In reference to a wave motion, define the terms (*i*) amplitude, (*ii*) time period, (*iii*) frequency, (*iv*) angular frequency, (*v*) wavelength, (*vi*) wave number, (*vii*) angular wave number and (*viii*) wave velocity.
3. Write Newton's formula for the speed of sound in gases. Why and what correction was applied by Laplace in this formula ? Also deduce modified formula for speed of sound. **[Delhi 06, 12]**
4. Derive Newton's formula for speed of sound in an ideal gas. What is Laplace correction ? **[Delhi 09]**
5. For a simple harmonic wave, deduce expressions for (*i*) particle velocity and (*ii*) particle acceleration. Write their phase relation with displacement.
6. What are stationary waves ? Explain the formation of stationary waves graphically. **[Chandigarh 04]**
7. Obtain an expression for a stationary wave formed by two sinusoidal waves travelling along the same path in opposite directions and obtain the positions of nodes and antinodes. **[Delhi 10]**
8. What are standing waves ? Derive an expression for the standing waves. Also define the terms node and antinode. **[Delhi 09]**
9. Discuss the formation of standing waves in a string fixed at both ends and the different modes of vibrations. **[Himachal 05C, 08C]**
10. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4. **[Delhi 06]**
11. What are beats ? Explain their formation analytically. Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves. **[Himachal 03]**
12. (*a*) What are beats ? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing waves.
    (*b*) Draw the fundamental modes of vibration of stationary waves in :
    (*i*) Closed pipe (*ii*) An open pipe. **[Central Schools 08]**

13. What is Doppler Effect in sound ? Obtain an expression for the observed frequency of sound produced by a source when both observer and source are in motion and medium is at rest. **[Delhi 12]**

14. (*a*) What is Doppler effect ?
(*b*) Derive an expression for the apparent frequency when a source moves towards a stationary observer.
(*c*) A policeman on duty detects a drop of 15% in the pitch of the horn of a motor-car as it crosses him. Calculate the speed of car, if the velocity of sound is 330 m/s. **[Central Schools 04, 07]**

Answers

1. Refer answer to Q. 3 on page 15.2.
2. Refer answer to Q. 7 on page 15.4.
3. Refer answer to Q. 11 on page 15.9.
4. Refer answer to Q.11 on page 15.9.
5. Refer answer to Q. 15 on page 15.16.
6. Refer answer to Q. 22 on page 15.25.
7. Refer answer to Q. 23 on page 15.26.
8. Refer answer to Q.26 on page 15.28.
9. Refer answer to Q. 26 on page 15.28.
10. Refer answer to Q. 27 on page 15.29.
11. Refer answer to Q. 34 and Q. 36 on page 15.41.
12. (*a*) Refer answer to Q.27 on page 15.29.
(*b*) (*i*) see Fig. 15.21(*a*) (*ii*) see Fig. 15.20(*a*).
13. Refer answer to Q. 42 on page 15.50.
14. Refer answer to Q.41 on page 15.49 and see hint of Problem 10 on page 15.57.

Competition Section

Waves

GLIMPSES

1. **Wave motion.** It is a kind of disturbance which travels through a medium due to the repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. In a wave both information and energy propagate from one point to another but there is no motion of matter as a whole through a medium.

2. **Three basic types of waves.**

   (*i*) ***Mechanical waves.*** The waves which require a mechanical medium for their propagation are called mechanical waves or elastic waves. For their propagation, the medium must possess the properties of inertia and elasticity. For example, water waves, sound waves, etc.

   (*ii*) ***Electromagnetic waves.*** The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation. For example, visible light, radio waves, etc.

   (*iii*) ***Matter waves.*** The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves. Matter waves associated with fast moving electrons are used in electron microscopes.

3. **Spring-model for the propagation of a wave through an elastic medium.** Energy transfer takes place because of the coupling through elastic forces between neighbouring oscillating parts of the medium.

4. **Transverse waves.** These are the waves in which particles of the medium vibrate about their mean positions in a direction perpendicular to the direction of propagation of the disturbance. These waves can propagate in those media which have a shear modulus of elasticity *e.g.*, solids.

5. **Longitudinal waves.** These are the waves in which particles of the medium vibrate about their mean positions along the direction of propagation of the disturbance. These waves can propagate in those media having a bulk modulus of elasticity and are therefore possible in all media : solids, liquids and gases.

6. **Progressive wave.** A wave that moves from one point of medium to another is called a progressive wave.

7. **Amplitude (A).** It is the maximum displacement suffered by the particles of the medium from the mean position during the propagation of a wave.

8. **Time period (T).** It is the time in which a particle of the medium completes one vibration about its mean position.

9. **Frequency (ν).** It is the number of waves produced per second in a given medium.

10. **Wavelength (λ).** It is the distance covered by a wave during the time a particle of the medium completes one vibration about its mean position. It is the distance between two nearest particles of the medium which are vibrating in the same phase.

11. **Angular wave number or propagation constant.** It represents the phase change per unit distance (or per unit path difference). It is equal to $2\pi/\lambda$.

    Thus $$k = \frac{2\pi}{\lambda}$$

    The SI unit of k is radian per metre (rad m^{-1}).

12. **Wave velocity (v).** It is the distance travelled by a wave in one second.

13. **Relation between wave velocity, frequency and wavelength.**

    Wave velocity = Frequency × wavelength

    or $$v = \nu\lambda$$

14. Relation between wave velocity, time period and wavelength.

$$\text{Wave velocity} = \frac{\text{Wavelength}}{\text{Time period}}$$

or
$$v = \frac{\lambda}{T} = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}.$$

15. Velocity of transverse waves.

(*i*) Velocity of transverse waves in a solid of modulus of rigidity η and density ρ is given by $v = \sqrt{\frac{\eta}{\rho}}$

(*ii*) Velocity of transverse waves in a string of mass per unit length m and stretched under tension T is given by $v = \sqrt{\frac{T}{m}}$

16. Velocity of longitudinal waves.

(*i*) Velocity of longitudinal waves in an extended solid (earth's crust) of bulk modulus κ, modulus of rigidity η and density ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

The factor $\kappa + (\frac{4}{3})\eta$ is called elongational elasticity.

(*ii*) Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

(*iii*) Velocity of longitudinal waves in a liquid of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

(*iv*) Velocity of longitudinal waves in a gaseous medium of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

According to Newton, when sound travels through gas, the changes taking place in the medium are isothermal in nature. So *Newton's formula* for the speed of sound is

$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$, where P = pressure of the gas.

According to Laplace, when sound travels in a gas, the changes taking place in the medium are adiabatic. So *Laplace formula* for the speed of sound is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

where $\gamma = \frac{C_p}{C_v}$ = specific heat ratio.

17. Factors affecting velocity of sound through gases

(*i*) *Effect of pressure.* Pressure has no effect on the speed of sound in a gas.

(*ii*) *Effect of density.* $v \propto \frac{1}{\sqrt{\rho}}$ or $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

(*iii*) *Effect of temperature.* As $v = \sqrt{\frac{\gamma RT}{M}}$

$\therefore \quad v \propto \sqrt{T}$ or $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

Temperature coefficient of velocity of sound is given by

$$\alpha = \frac{v_t - v_0}{t}$$

For air, $\alpha = 0.61 \text{ ms}^{-1}\,^\circ\text{C}^{-1}$.

(*iv*) *Effect of humidity.* Sound travels faster in moist air.

(*v*) *Effect of wind.* If the wind blows with velocity w in a direction making an angle θ with the direction of sound, then the resultant velocity of sound will be $v' = v + w\cos\theta$

18. Wave equation. A plane progressive harmonic wave travelling along positive X-direction may be represented as

(*i*) $y = A\sin(\omega t - kx)$, where $k = 2\pi/\lambda$

(*ii*) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

(*iii*) $y = A\sin\frac{2\pi}{T}(vt - x)$

where v is the wave-velocity, λ the wavelength, A the amplitude of the oscillating particles of the medium and y is the displacement of the particle located at position x at any instant t.

If the wave is travelling along *negative* X-direction, the minus sign is replaced by plus sign in the above equations. Thus

(*i*) $y = A\sin(\omega t + kx)$, where $k = 2\pi/\lambda$

(*ii*) $y = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$

(*iii*) $y = A\sin\frac{2\pi}{\lambda}(vt + x)$

19. Phase and phase difference. Phase is the argument of the sine or cosine function representing the wave. Thus phase,

$$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

The relation between phase difference (Δϕ) and time interval Δt is

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

Thus in time period T, the phase of a particle changes by 2π.

The relation between phase difference ($\Delta\phi$) and path difference (Δx) is

$$\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that farther the particle is located from the origin in the positive X-direction, the more it lags behind in phase. Clearly, the phase difference between two particles located at separation λ is 2π.

20. **Principle of superposition of waves.** When a number of waves travel through a medium simultaneously, the resultant displacement at any point of the medium is equal to the vector sum of the displacements of the individual waves. If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ..., \vec{y}_n$ are the displacements of n waves superposing each other at a point, then the resultant displacement at that point will be

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ... + \vec{y}_n .$$

21. **Reflection of a wave.** When a wave is reflected from a rigid boundary or a closed end, it is reflected back with a phase reversal or phase difference of π radians but reflection at an open boundary takes place without any phase change.

22. **Stationary waves.** When two progressive waves of equal amplitude and frequency, travelling in opposite directions along a straight line superpose each other, the resultant wave does not travel in either direction and is called a *stationary* or *standing wave*. At some points, the particles of the medium always remain at rest. These are called *nodes*. At some other points, the amplitude of oscillation is maximum. These are called *antinodes*.

Consider a plane progressive harmonic wave travelling along positive X-direction.

$$y_1 = A\sin(\omega t - kx) \qquad \text{(incident wave)}$$

If this wave is reflected from a *free end*, then

$$y_2 = A\sin(\omega t + kx) \qquad \text{(reflected wave)}$$

The stationary wave formed by the superposition of the incident and reflected waves will be

$$y = y_1 + y_2 = 2A\cos kx \sin\omega t$$

In this case, the points $x = 0, \lambda/2, 3\lambda/2, ...$ will be antinodes and the points $x = \lambda/4, 3\lambda/4, 5\lambda/4$, will be nodes.

If the wave is reflected from a *rigid end*, then

$$y_2 = -A\sin(\omega t + kx)$$

The equation of the stationary wave will be

$$y = -2A\sin kx\cos\omega t$$

In this case, the points $x = 0, \lambda/2, \lambda, 3\lambda/2, ...$, will be nodes and the points $x = \lambda/4, 3\lambda/4, 5\lambda/4, ...$ will be antinodes.

Separation between two successive nodes or antinodes $= \lambda/2$

Separation between a node and nearest antinode $= \lambda/4$

23. **Modes of vibrations of strings.** On a stretched string, transverse stationary waves are formed due to superposition of direct and the reflected transverse waves.

For *fundamental mode* :

$$\lambda_1 = 2L, \quad \nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \text{ (say)}$$

For *second mode* :

$$\lambda_2 = L, \quad \nu_2 = 2\nu$$

(*second harmonic* or *first overtone*)

For *third mode* :

$$\lambda_3 = 2L/3, \quad \nu_3 = 3\nu$$

(*third harmonic or second overtone*)

For *pth mode* : When the string vibrates in p loops,

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu$$

[*pth harmonic* or *$(p-1)$th overtone*]

Fundamental frequency for a string of diameter D and density ρ,

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}} .$$

24. **Laws of transverse vibrations of a stretched string.** The fundamental frequency of a vibrating is

(*i*) inversely proportional to its length.

$$\nu \propto \frac{1}{L} \qquad \text{(Law of length)}$$

(*ii*) directly proportional to the square root of its tension.

$$\nu \propto \sqrt{T} \qquad \text{(Law of tension)}$$

(*iii*) inversely proportional to its mass per unit length.

$$\nu \propto \frac{1}{\sqrt{m}} \qquad \text{(Law of mass)}$$

Combining all the factors, we get

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

where ρ is the density and D the diameter of the string.

25. **Organ pipe.** It is the simplest musical instrument in which sound is produced by setting an air column into vibrations. Longitudinal stationary waves are formed on account of superposition of incident and reflected longitudinal waves.

26. Modes of vibrations of closed organ pipes. Longitudinal stationary waves are formed in an organ pipe closed at one end.

For *fundamental mode*

$$\lambda_1 = 4L, \quad \nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

For *second mode* : $\lambda_2 = 4L/3, \quad \nu_2 = 3\nu$

(*third harmonic* or *first overtone*)

For *third mode* : $\lambda_3 = 4L/5, \ \nu_3 = 5\nu$

(*fifth harmonic* or *second overtone*)

For *pth mode* :

$$\lambda_p = 4L/(2p-1), \quad \lambda_p = (2p-1)\,\nu$$

[(2 p − 1) *harmonic or* (p − 1)*th overtone*]

Here $\nu_1 : \nu_2 : \nu_3 : \nu_4 \ldots = 1:3:5:7:\ldots$

(*only odd harmonics*)

27. Modes of vibrations of open organ pipe. Antinodes are formed at both ends, separated by one node in the fundamental mode.

For *fundamental mode*:

$$\lambda_1' = 2L, \ \nu_1' = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

For *second mode* : $\lambda_2' = L, \quad \nu_2' = 2\nu$

(*second harmonic or first overtone*)

For *third mode* : $\lambda_3 = 2L/3, \quad \nu_3 = 3\nu$

(*third harmonic or second overtone*)

For *pth mode* $\lambda_p' = 2L/p, \quad \nu_p' = p\nu$

[*pth harmonic or* (p − 1) *overtone*]

Here $\nu_1 : \nu_2 : \nu_3 : \nu_4 : \ldots = 1:2:3:4:\ldots$

(*Both odd and even harmonics*)

28. Resonance tube. It is an organ pipe closed at one end. If L_1 and L_2 be the first and second resonance lengths with a tuning fork of frequency ν, then the speed of sound in air is given by

$$v = 4\nu\,(L_1 + 0.3\,D),$$

D = internal diameter of resonance tube

or $\quad v = 2\nu\,(L_2 - L_1)$

End correction $= 0.3\,D = \dfrac{L_2 - 3L_1}{2}$.

29. Vibrations in rods clamped in the middle. Here antinodes are formed at the ends and a node in the middle.

Fundamental frequency, $\quad \nu_1 = \dfrac{v}{2L} = \nu$

First overtone or third harmonic, $\nu_2 = \dfrac{3v}{2L} = 3\nu$

Second overtone or fifth harmonic, $\nu_3 = \dfrac{5v}{2L} = 5\nu$

30. Beats. The periodic variations in the intensity of sound due to the superposition of two sound waves of slightly different frequencies are called beats. One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.

Beat frequency, $\nu_{beat} = \nu_1 - \nu_2$

Beats may be used to determine the frequency of a tuning fork. It must be noted here that

(*i*) When the prong of a tuning fork is slightly loaded with wax, its frequency of vibration decreases.

(*ii*) When the prong of a tuning fork is filed slightly, its frequency of vibration increases.

31. Doppler effect in sound. The phenomenon of the change in apparent pitch of sound due to relative motion between the source of sound and the observer is called Dopper effect. If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium (in the direction of sound) respectively, then the apparent frequency is given by

$$\nu' = \frac{v + v_m - v_0}{v + v_m - v_s} \times \nu$$

For the medium at rest ($v_m = 0$),

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

Here the velocities are taken *positive* in the *source to observer* ($S \rightarrow O$) direction and *negative* in the opposite ($O \rightarrow S$) direction.

Special cases :

(*i*) When the source moves towards the stationary observer,

$$\nu' = \frac{v}{v - v_s} \times \nu \qquad (\nu' > \nu)$$

(*ii*) When the source moves away from the stationary observer,

$$\nu' = \frac{v}{v + v_s} \times \nu \qquad (\nu' < \nu)$$

(*iii*) When the observer moves towards the stationary source,

$$\nu' = \frac{v + v_0}{v} \times \nu \qquad (\nu' > \nu)$$

(*iv*) When the observer moves away from the stationary source,

$$\nu' = \frac{v - v_0}{v} \times \nu \qquad (\nu' < \nu)$$

(*v*) When both source and observer move towards each other,

$$\nu' = \frac{v + v_0}{v - v_s} \times \nu \qquad (\nu' > \nu)$$

(*vi*) When both source and observer move away from each other,

$$\nu' = \frac{v - v_0}{v + v_s} \times \nu \qquad (\nu' < \nu)$$

(*vii*) When source moves towards observer and observer away from the source,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

(*viii*) When source moves away from observer and observer towards the source,

$$\nu' = \frac{v + v_0}{v + v_s} \times \nu$$

32. Musical sound. A musical sound consists of quick, regular and periodic succession of compressions and rarefactions without a sudden change in amplitude. It produces pleasing effect on the ears.

33. Noise. A noise consists of slow, irregular and non periodic succession of compressions and rarefractions, that may have a sudden change in amplitude. It produces non-pleasing effect on the ears.

34. Intensity of sound (I). The intensity of sound at any point may be defined as the amount of sound energy passing per unit time per unit area around that point in a perpendicular direction.

35. Zero level or threshold of hearing (I_0). The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing.

For a sound of frequency 1 kHz, it is found that the threshold of hearing is 10^{-12} Wm^{-2}.

36. Characteristics of a musical sound. These are

(*i*) ***Pitch.*** It is the characteristic of musical sound that helps the listener in distinguishing a shrill note from a grave (flat or dull) one. It depends on frequency.

(*ii*) ***Quality.*** It is the characteristic of the musical sound that distinguishes between two sounds of same pitch and loudness from one another. It depends on the number or intensity of overtones.

(*iii*) ***Loudness.*** The sensation of hearing which enables us to distinguish between a loud and a faint sound, is called loudness. It depends on intensity.

37. Units of loudness. The unit of loudness of a sound is ***bel***. The loudness of a sound is said to be 1 bel, if its intensity is 10 times that of the threshold of hearing.

The loudness of a sound of intensity I is given by

$$L = \log_{10} \frac{I}{I_0},$$

where I_0 is threshold of hearing. It is called *Weber-Fechner law*.

The practical and a smaller unit of loudness is ***decibel*** **(dB)**.

$$1 \text{ decibel} = \frac{1}{10} \text{ bel}$$

In decibels, the loudness of a sound of intensity I is given by

$$L = 10 \log_{10} \frac{I}{I_0}$$

38. Reverberation of sound. It is the phenomenon of persistence of sound after the source has stopped producing sound. The time for which sound persists after the source has stopped producing sound is called *reverberation time* (T).

According to *Sabine* law, reverberation time of a hall is given by

$$T = \frac{0.16\, V}{\Sigma\, a_i\, s_i}$$

where V is volume of the hall and $\Sigma\, a_i\, s_i = a_1 s_1 + a_2 s_2 + \ldots$ is the total absorption of the hall. Here $s_1, s_2 \ldots$ are the surface areas of materials with absorption coefficients $a_1, a_2, \ldots$ respectively.

39. Interval between two notes. The ratio of the frequencies of two notes is called interval between them. Two notes are said to be in *unison* if their frequencies are equal, that is interval between them is 1 : 1. Some other common intervals found useful in producing musical sound are as follows :

(*a*) *octave* (1 : 2) (*b*) *majortone* (8 : 9)

(*c*) *minortone* (9 : 10) (*d*) *semitone* (15 : 16)

The interval between any two notes is obtained by multiplying the various intervening intervals.

40. Musical scale. A series of notes arranged such that their fundamental frequencies have definite ratios is called a musical scale.

41. Major diatonic scale. The eight notes of major diatonic scale starting with 256 as keynote along with their Helmholtz and Indian notation are as below :

Helmholtz notation :	C	D	E	F	G	A	B	C
Indian notation :	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
Frequency of notes with 256 as keynote :	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
Intervals :		9/8	10/9	16/15	9/8	10/9	9/8	16/15

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer, who is standing in air, is

(*a*) 200 Hz (*b*) 3000 Hz
(*c*) 120 Hz (*d*) 600 Hz. [IIT 04]

2. The ratio of the speed of sound in nitrogen gas to that in the helium gas at 300 K is

(*a*) $\sqrt{2/7}$ (*b*) $\sqrt{1/7}$
(*c*) $\sqrt{3}/5$ (*d*) $\sqrt{6}/5$ [IIT 99]

3. Two monatomic ideal gases 1 and 2 of molecular masses M_1 and M_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

(*a*) $\sqrt{\frac{M_1}{M_2}}$ (*b*) $\sqrt{\frac{M_2}{M_1}}$
(*c*) $\frac{M_1}{M_2}$ (*d*) $\frac{M_2}{M_1}$. [IIT 2K]

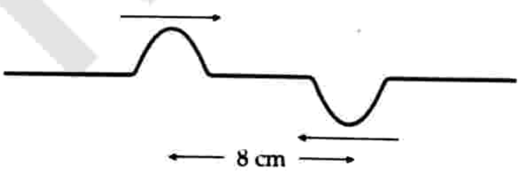
4. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

(*a*) $1.22v$ (*b*) $0.61v$
(*c*) $1.50v$ (*d*) $0.75v$. [IIT 80]

5. A travelling wave in a stretched string is described by the equation $y = A\sin(kx - \omega t)$. The maximum particle velocity is

(*a*) $A\omega$ (*b*) ω/k
(*c*) $d\omega/dk$ (*d*) x/t. [IIT 97]

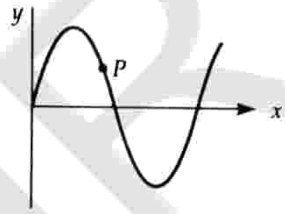
6. Two pulses in a stretched string whose centres are intially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is



2 cm/s. After 2 seconds, the total energy of the pulses will be

(*a*) zero (*b*) purely kinetic
(*c*) purely potential
(*d*) purely kinetic and partly potential. [IIT 2K]

7. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is



(*a*) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (*b*) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s
(*c*) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (*d*) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s [IIT 08]

8. A wave represented by the equation

$$y = a\cos(kx - \omega t)$$

is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is

(*a*) $a\cos(kx - \omega t)$ (*b*) $-a\cos(kx - \omega t)$
(*c*) $-a\cos(kx + \omega t)$ (*d*) $-a\sin(kx - \omega t)$ [IIT 88]

9. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1/f_2 is given by

(*a*) 2 (*b*) 4
(*c*) 8 (*d*) 1 [IIT 2K]

10. A sonometer wire resonates with a given tuning form forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning form forming three antinodes for the same positions of the bridges. The value of M is

(*a*) 25 kg (*b*) 5 kg
(*c*) 12.5 kg (*d*) (1/25) kg [IIT 02]

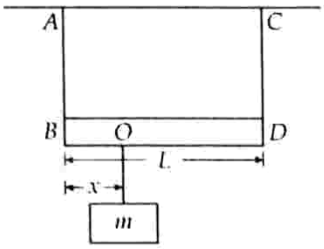
11. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its

volume is submerged. The new fundamental frequency in Hz is

(*a*) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (*b*) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(*c*) $300\left(\frac{2\rho}{2\rho-1}\right)$ (*d*) $300\left(\frac{2\rho-1}{2\rho}\right)$ [IIT 95]

12. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is



equal to x. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. x is

(*a*) $\frac{L}{5}$ (*b*) $\frac{4L}{5}$

(*c*) $\frac{3L}{5}$ (*d*) $\frac{L}{4}$. [IIT 06]

13. In the experiment to determine the speed of sound using a resonance column,

(*a*) prongs of the tuning fork are kept in a vertical plane

(*b*) prongs of the tuning fork are kept in a horizontal plane

(*c*) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air

(*d*) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air. [IIT 07]

14. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option

(*a*) $n=3, f_2=\frac{3}{4}f_1$ (*b*) $n=3, f_2=\frac{5}{4}f_1$

(*c*) $n=5, f_2=\frac{3}{4}f_1$ (*d*) $n=5, f_2=\frac{5}{4}f_1$. [IIT 05]

15. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is

(*a*) 51.2 cm/s (*b*) 102.4 m/s

(*c*) 204.8 cm/s (*d*) 153.6 cm/s. [IIT 96]

16. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

(*a*) 200 Hz (*b*) 300 Hz

(*c*) 240 Hz (*d*) 480 Hz. [IIT 96]

17. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

(*a*) 0.012 m (*b*) 0.025 m

(*c*) 0.05 m (*d*) 0.024 m. [IIT 03]

18. A pipe of length l_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal.

(*a*) $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (*b*) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$

(*c*) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (*d*) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$ [IIT 04]

19. The ends of a stretched wire of length L are fixed at $x=0$ and $x=L$. In one experiment, the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then

(*a*) $E_2 = E_1$ (*b*) $E_2 = 2E_1$

(*c*) $E_2 = 4E_1$ (*d*) $E_2 = 16E_1$ [IIT 01]

20. Two plane harmonic sound waves are expressed by the equations :

$$y_1(x,t) = A\cos(0.5\pi x - 100\pi t)$$

and $$y_2(x,t) = A\cos(0.46\pi x - 92\pi t)$$

All parameters are in mks system. How many times does an observer hear maximum intensity in one second ?

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 10 [IIT 06]

21. A vibrating string of length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to 340 m/s, the frequency n of the tuning fork in Hz is

(*a*) 344 (*b*) 336

(*c*) 117.3 (*d*) 109.3 [IIT 08]

22. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)

(*a*) 409 (*b*) 429

(*c*) 517 (*d*) 500 [IIT 97]

23. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1 / f_2 is

(*a*) $\frac{18}{19}$ (*b*) $\frac{1}{2}$

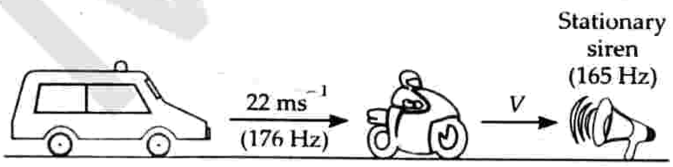
(*c*) 2 (*d*) $\frac{19}{18}$ [IIT 2K]

24. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is

(*a*) 242/252 (*b*) 2

(*c*) 5/6 (*d*) 11/6 [IIT 2K]

25. A police car moving at 22 ms^{-1}, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of



frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.

(*a*) 33 ms^{-1} (*b*) 22 ms^{-1}

(*c*) 11 ms^{-1} (*d*) zero. [IIT 03]

26. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

(*a*) 5 grams (*b*) 10 grams

(*c*) 20 grams (*d*) 40 grams [IIT 2010]

27. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(*a*) 8.50 kHz (*b*) 8.25 kHz

(*c*) 7.75 kHz (*d*) 7.50 kHz [IIT 2011]

Multiple Choice Questions with One or More Than One Correct Answer

28. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin(60t + 2x)$, where x and y are in metre and t is time in second. This represents a wave

(*a*) travelling with a velocity of 30 m/s in the negative x-direction

(*b*) of wavelength π m

(*c*) of frequency $(30/\pi)$ hertz

(*d*) of amplitude 10^{-4} m travelling along the negative x-direction. [IIT 82]

29. A transverse wave is described by the equation

$$y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right).$$

The maximum particles velocity is equal to four times the wave velocity if

(*a*) $\lambda = \pi \frac{y_0}{4}$ (*b*) $\lambda = \pi \frac{y_0}{2}$

(*c*) $\lambda = \pi y_0$ (*d*) $\lambda = 2\pi y_0$. [IIT 84]

30. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are

(*a*) $\cos kx \sin \omega t$ (*b*) $k^2x^2 - \omega^2 t^2$

(*c*) $\cos^2(kx + \omega t)$ (*d*) $\cos(k^2x^2 - \omega^2 t^2)$

[IIT 87]

31. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in metre and t is in second. The expression represents

(*a*) a wave travelling in the positive x-direction with a velocity 1.5 m/s

(*b*) a wave traelling in the negative x-direction with a velocity 1.5 m/s

(*c*) a wave travelling in the negative x-direction having a wavelength 0.2 m

(*d*) a wave travelling in the positive x-direction having a wavelength 0.2 m. [IIT 99]

32. In a wave motion $y = a\sin(kx - \omega t)$, y can represent

(*a*) electric field (*b*) magnetic field

(*c*) displacement (*d*) pressure. [IIT 99]

33. $y(x,t) = 0.8/[(4x+5t)^2+5]$ represents a moving pulse, where x and y are in metre and t in second.

Then

(*a*) pulse is moving in $+x$ direction

(*b*) in 2 s it will travel a distance of 2.5 m

(*c*) its maximum displacement is 0.16 m

(*d*) it is a symmetric pulse. [IIT 99]

34. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\,ms^{-1}$, then λ and f are given by

(*a*) $\lambda = 2\pi \times 10^{-2}$ m (*b*) $\lambda = 10^{-3}$ m

(*c*) $f = 10^3/2\pi$ Hz (*d*) $f = 10^4$ Hz. [IIT 98]

35. As a wave propagates,

(*a*) the wave intensity remains constant for a plane wave

(*b*) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(*c*) the wave intensity decreases as the the inverse square of the distance from the source for a spherical wave

(*d*) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all time. [IIT 99]

36. Standing waves can be produced

(*a*) on a string clamped at both the ends

(*b*) on a string clamped at one end free at the other

(*c*) when incident wave gets reflected from a wall

(*d*) when two identical waves with a phase difference of π are moving in the same direction.

[IIT 99]

37. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is

(*a*) 31.25 (*b*) 62.50

(*c*) 93.75 (*d*) 125. [IIT 85]

38. A tube closed at one end and containing air produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)

(*a*) 1024 (*b*) 512

(*c*) 256 (*d*) 128. [IIT 80]

39. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then

(*a*) the intensity of the sound heard at the first resonance was more than that at the second resonance

(*b*) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(*c*) the amplitude of the vibration of the ends of the prongs is typically around 1 cm

(*d*) the length of air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

[IIT 09]

40. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork . The ratio of the length of P_1 to that of P_2 is

(*a*) 8/3 (*b*) 3/8

(*c*) 1/6 (*d*) 1/3. [IIT 88]

41. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency

(*a*) 80 Hz (*b*) 240 Hz
(*c*) 320 Hz (*d*) 400 Hz.

[IIT 99]

42. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is

(*a*) 0.05 s (*b*) 0.10 s
(*c*) 0.20 s (*d*) 0.40 s. [IIT 98]

43. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tensions in the strings, then it could be said that while making the above changes in tension

(*a*) T_2 was decreased
(*b*) T_2 was increased
(*c*) T_1 was decreased
(*d*) T_1 was increased. [IIT 91]

44. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000\,t)$$

This expression may be considered to be a result of the superposition of

(*a*) two (*b*) three
(*c*) four (*d*) five.

independent harmonic motions. [IIT 92]

45. A wave disturbance in a medium is described by

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x),$$

where x and y are in metre and t is in second.

(*a*) A node occurs at $x = 0.15$ m
(*b*) An antinode occurs at $x = 0.3$ m
(*c*) The speed of wave is 5 ms^{-1}
(*d*) The wavelength is 0.2 m. [IIT 95]

46. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c.

(*a*) The number of waves striking the surface per second is $f\frac{(c+v)}{c}$
(*b*) The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$
(*c*) The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$
(*d*) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$.

[IIT 95]

47. The (x, y) co-ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expressions for u is (are) (a = positive constant)

(*a*) $a\cos(\pi x/2L)\cos(\pi y/2L)$
(*b*) $a\sin(\pi x/L)\sin(\pi y/L)$
(*c*) $a\sin(\pi x/L)\sin(2\pi y/L)$
(*d*) $a\cos(2\pi x/L)\sin(\pi y/L)$. [IIT 98]

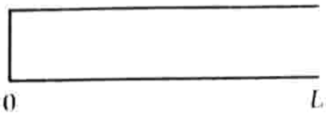
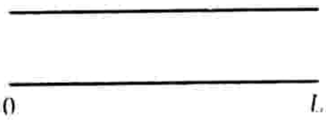
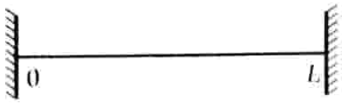
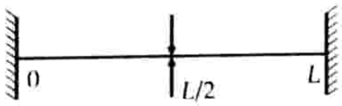
Integer Answer Type

48. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, what is the amplitude of the resultant wave ? [IIT 2010]

49. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. [IIT 2010]

Match-Matrix Type

50. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. [IIT 2011]

Column I	Column II
(a) Pipe closed at one end 	(p) Longitudinal waves
(b) Pipe open at both ends 	(q) Transverse waves
(c) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(d) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

Answers and Explanations

1. (*d*) The frequency of sound does not change during its refraction from water into air.

2. (*c*) $\dfrac{v_{N_2}}{v_{He}} = \sqrt{\dfrac{\gamma_{N_2}}{\gamma_{He}} \cdot \dfrac{M_{He}}{M_{N_2}}} = \sqrt{\dfrac{7/5}{5/3} \cdot \dfrac{4}{28}} = \dfrac{\sqrt{3}}{5}$.

3. (*b*) As $v = \sqrt{\dfrac{3RT}{M}}$ *i.e.*, $v \propto \dfrac{1}{\sqrt{M}}$

$\therefore \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{M_2}{M_1}}$.

4. (*c*) According to Hooke's law,

Tension (T) $\propto$ Extension (x)

Also, speed of sound,

$v \propto \sqrt{T} \qquad \therefore \qquad v \propto \sqrt{x}$

$v' \propto \sqrt{1.5x}$

or $\quad \dfrac{v'}{v} = \sqrt{1.5} = 1.22$

or $\quad v' = 1.22\, v$.

5. (*a*) $\quad y = A \sin(kx - \omega t)$

$u = \dfrac{dy}{dx} = -\omega A \cos(kx - \omega t)$

$\therefore \quad u_{max} = \omega A$.

6. (*b*) After 2 s, crest moves 4 cm towards right and trough moves 4 cm towards left. They superpose and cancel the displacement of each other. The string becomes straight. The energy becomes totally kinetic.

7. (*a*) Here $v = 10\ ms^{-1}$, $\lambda = 0.5\ m = 50\ cm$, $A = 10\ cm$, $y = 5\ cm$.

Particle velocity,

$$u = \omega\sqrt{A^2 - y^2} = \frac{2\pi v}{\lambda}\sqrt{A^2 - y^2}$$

$$= \frac{2\pi \times 10}{50}\sqrt{10^2 - 5^2}\ cms^{-1}$$

$$= 2\sqrt{3}\,\pi\ cms^{-1}$$

$$= \frac{\sqrt{3}\pi}{50}\ ms^{-1}$$ **(in +ve y-direction).**

8. (*c*) For the formation of a stationary wave, two identical waves travelling in opposite directions must superpose each other. At $x = 0$, resultant y should be zero for getting a node. Hence option (*c*) is correct.

$$y = y_1 + y_2 = a\cos(kx - \omega t) - a\cos(kx + \omega t)$$
$$= 2a \sin kx \sin \omega t$$

At $x = 0$, $y = 0$ *i.e.*, a node is formed.

9. (*d*) $f = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{\pi R^2 \rho}} = \frac{1}{2lR}\sqrt{\frac{T}{\pi\rho}}$

As T and ρ are same for both strings,

$\therefore \quad f \propto \frac{1}{lR}$

For first string, $f_1 \propto \frac{1}{L \times 2r}$

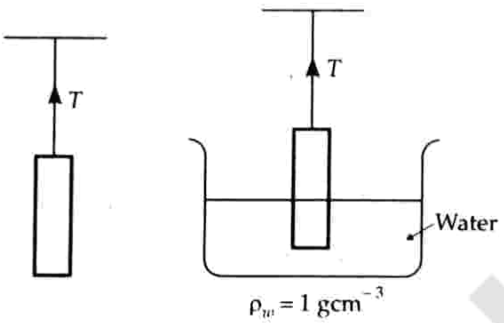
For second string, $f_2 \propto \frac{1}{2L \times r}$

Hence $f_1 = f_2$ or $\frac{f_1}{f_2} = \mathbf{1}$.

10. (*a*) $M = 25$ kg.

Refer to the solution of Problem 10 on page 15.69.

11. (*a*) The situation is shown in the figure.



In air, $T = Mg = V\rho g$

$$\therefore \quad \nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{V\rho g}{m}}$$

When the object is half immersed in water

$$T' = Mg - \text{upthrust}$$

$$= V\rho g - \left(\frac{V}{2}\right)\rho_\omega g$$

$$= \left(\frac{V}{2}\right)g(2\rho - \rho_\omega)$$

New fundamental frequency,

$$\nu' = \frac{1}{2l}\sqrt{\frac{T'}{m}} = \frac{1}{2l}\sqrt{\frac{(V/2)g(2\rho-\rho_\omega)}{m}}$$

$$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\frac{2\rho-\rho_\omega}{2\rho}}$$

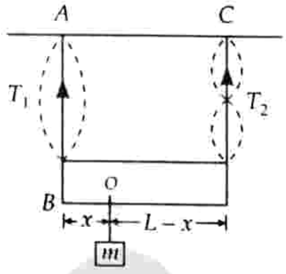
or

$$\nu' = \nu\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2}$$

$$= 300\left(\frac{2\rho-1}{2\rho}\right)^{1/2} \text{ Hz.}$$

12. (*a*) Frequency of first harmonic in AB

= Frequency of second harmonic in CD



$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_1}{m}} = \frac{1}{l}\sqrt{\frac{T_2}{m}}$$

or $\quad T_1 = 4T_2 \quad ...(i)$

For translational equilibrium,

$$T_1 + T_2 = mg \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$T_1 = \frac{4mg}{5} \text{ and } T_2 = \frac{mg}{5}$$

For rotational equilibrium about O,

Torque due to T_1 about O

= Torque due to T_2 about O

$$T_1 x = T_2(L-x)$$

or $\quad \frac{4mg}{5}x = \frac{mg}{5}(L-x)$

or $\quad 4x = L - x$ or $x = \frac{\mathbf{L}}{\mathbf{5}}$.

13. (*a*) The prongs of vibrating tuning fork are kept in a vertical plane just above opening of the resonance tube.

14. (*d*) In case of open pipe, frequency of second harmonic is

$$f_1 = \frac{v}{\lambda} = \frac{v}{L} \qquad [\lambda = L]$$

In case of closed pipe, frequency of nth harmonic,

$$f_2 = \frac{nv}{4L}$$

where n is odd number.

Clearly, $f_2 = \frac{n}{4}f_1$.

If $n = 3$, $f_2 < f_1$, which is not acceptable.

If $n = 5$, $f_2 = \frac{5}{4}f_1$, which is acceptable.

Hence option (*d*) is correct.

15. (*c*) Speed of sound,

$$v = 2\nu(l_2 - l_1)$$

Maximum possible error in speed,

$$\Delta v = 2\nu(\Delta l_2 + \Delta l_1)$$

$$= 2 \times 512(0.1 + 0.1) = \mathbf{204.8 \ cm \ s^{-1}}.$$

16. (*a*) $\nu = 200$ Hz.

Refer to the solution of Problem 18 on page 15.70.

17. (*b*) End correction $= \frac{L_2 - 3L_1}{2}$

$$= \frac{0.35 - 3 \times 0.1}{2} = \mathbf{0.025 \ m}.$$

18. (*b*) f_c (first overtone) = f_0 (first over tone)

$$3\left(\frac{v_c}{4l_1}\right) = 2\left(\frac{v_0}{2l_0}\right)$$

$$l_0 = \frac{4}{3}\left(\frac{v_0}{v_c}\right)l_1$$

As $\quad v \propto \frac{1}{\sqrt{\rho}} \quad$ or $\quad \frac{v_0}{v_c} = \sqrt{\frac{\rho_1}{\rho_2}}$

$$\therefore \quad l_2 = \frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}.$$

19. (*c*) $E_2 = 4E_1$, refer to the solution of Problem 9 on page 15.69.

20. (*a*) For first wave,

$$\omega_1 = 2\pi\nu_1 = 100\,\pi$$

$$\therefore \quad \nu_1 = 50 \text{ Hz}$$

For second wave,

$$\omega_2 = 2\pi\nu_2 = 92\,\pi$$

$$\therefore \quad \nu_2 = 46 \text{ Hz}$$

Beat frequency $= \nu_1 - \nu_2 = 4$ Hz.

Hence the intensity of sound becomes maximum 4 times in one second.

21. (*a*) If ν is the frequency of the string, then

$$n - 4 = \nu$$

Third harmonic of closed pipe,

$$\nu = \frac{3v}{4L}$$

where L = Length of the tube, and

v = velocity of sound in air

$$\therefore \quad n = \nu + 4 = \frac{3v}{4L} + 4 = \frac{3\times 340}{4\times 0.75} + 4$$

$$= \mathbf{344\ Hz.}$$

22. (*d*) $\quad \nu' = \frac{v}{v - v_s} \times v = \frac{330}{330 - 33} \times 450$

$$= \mathbf{500\ Hz.}$$

23. (*d*) $\quad \frac{f_1}{f_2} = \frac{19}{18}$

Refer to the solution of Problem 12 on page 15.69.

24. (*b*) $\frac{v_B}{v_A} = 2$

Refer to the solution of Problem 13 on page 15.69.

25. (*b*) $v = 22$ ms^{-1}.

Refer to the solution of Problem 14 on page 15.70.

26. (*b*) For hollow pipe, fundamental frequency is

$$f = \frac{v}{4l} = \frac{320}{4\times 0.8}$$

For string in 2nd harmonic,

$$f = \frac{1}{l}\sqrt{\frac{T}{\mu}} = \frac{1}{l}\sqrt{\frac{Tl}{m}} = \frac{1}{0.5}\sqrt{\frac{50\times 0.5}{m}}$$

Equating and solving, we get

$$m = 0.01 \text{ kg} = \mathbf{10\ g}$$

27. (*a*) Frequency received by the building

$$\nu' = \left(\frac{v}{v - v_c}\right)\nu$$

The wall (source) reflects this frequency, so frequency heard by the car driver is

$$\nu'' = \left(\frac{v + v_c}{v}\right)\nu' = \left(\frac{v + v_c}{v}\right)\left(\frac{v}{v - v_c}\right)\nu$$

$$= \left(\frac{v + v_c}{v - v_c}\right)\nu$$

$$= \left(\frac{320 + 10}{320 - 10}\right) \times 8 \text{ kHz}$$

$$[v_c = 36 \text{ kmh}^{-1} = 10 \text{ ms}^{-1}]$$

$$= \frac{33}{31} \times 8 = \mathbf{8.5\ kHz}$$

28. (*a*), (*b*), (*c*), (*d*)

$$y = 10^{-4}\sin(60t + 2x)$$

Standard equation of a wave travelling along –ve *x*-direction is

$$y = a\sin(\omega t + kx)$$

$$\therefore \quad a = 10^{-4} \text{ m}, \quad \omega = 60 \text{ rad s}^{-1}$$

$$\nu = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi} \text{ Hz.}$$

$$k = \frac{2\pi}{\lambda} = 2 \text{ rad m}^{-1}$$

$$v = \frac{\omega}{k} = \frac{60}{2} = 30 \text{ ms}^{-1}$$

Clearly, all the given options are correct.

29. (*b*) $\quad y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

$$u = \frac{dy}{dt} = 2\pi f y_0 \cos 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$u_{max} = 2\pi f y_0$$

Wave velocity,

$$v = f\lambda$$

$$\therefore \quad 2\pi f y_0 = 4 \times f\lambda$$

or $$\lambda = \frac{\pi y_0}{2}.$$

30. (*a*) $y = \cos kx \sin \omega t$ is the only equation of standing wave. The conditions for a function of x and t to represent a wave is

$$\frac{\partial^2 y}{\partial x^2} = \text{constant} \times \frac{\partial^2 y}{\partial t^2}$$

Only first expression satisfies this condition.

31. (*b*), (*c*)

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

$$y = A\sin(kx + \omega t + \phi)$$

This is the equation of wave travelling along –ve x-direction. Hence options (*a*) and (*d*) are incorrect.

$$k = \frac{2\pi}{\lambda} = 10\pi, \quad \omega = 15\pi$$

$$\lambda = \frac{1}{5} = 0.2 \text{ m}$$

$$v = \frac{\omega}{k} = \frac{15\pi}{10\pi} = 1.5 \text{ ms}^{-1}$$

Hence options (*b*) and (*c*) are correct.

32. (*a*), (*b*), (*c*), (*d*) $y = a\sin(kx - \omega t)$

(*a*) y represents electric field in an electromagnetic wave.

(*b*) y represents magnetic field in an electromagnetic wave.

(*c*) y represents displacement in a sound wave

(*d*) y represents pressure in a sound wave.

33. (*b*), (*c*), (*d*) A wave produced by a sudden disturbance of short duration is called a pulse. Its shape does not change during propagation. It can be expressed as

$$y = \frac{a}{b + (x \mp vt)^2}$$

–ve sign for propagation along +ve x-direction and +ve sign for propagation along –ve x-direction.

Given : $$y(x,t) = \frac{0.8}{(4x+5t)^2 + 5}$$

$$= \frac{0.8}{5 + 16[x + (5/4)t]^2}$$

We can note that,

(*a*) The pulse is moving along –ve x-direction. Hence option (*a*) is incorrect.

(*b*) $vt = (5/4)t$ or $v = 1.25 \text{ ms}^{-1}$

Distance travelled in 2 s $= 1.25 \times 2 =$ **2.5 m.**

Hence option (*b*) is correct.

(*c*) Put $(4x+5t)^2 = 0$, then

$$y_{max} = \frac{0.8}{5} = \mathbf{0.16 \text{ m}}.$$

Hence option (*c*) is correct.

(*d*) At $t = 0$,

$$y(x) = \frac{0.8}{16x^2 + 5}$$

Also, $$y(-x) = \frac{0.8}{16x^2 + 5}$$

i.e., $$y(x) = y(-x) \text{ at } t = 0.$$

Thus the given pulse is symmetric. Hence option (*d*) is correct.

34. (*a*), (*c*) $\lambda = 2\pi \times 10^{-2}$ m, $f = 10^3/2\pi$ Hz.

Refer to the solution of Problem 17 on page 15.70.

35. (*a*), (*c*), (*d*) For a spherical wave, $I \propto 1/r^2$. With the increase in distance from the source, though intensity decreases, the total energy transmitted remains the same.

36. (*a*), (*b*), (*c*) Standing waves can be formed by superposition of the identical waves (same frequency and wavelength) travelling in opposite directions. Hence options (*a*), (*b*) and (*c*) are correct. Option (*d*) is not correct as the waves are not travelling in opposite directions.

37. (*a*), (*c*) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L}$$

$$\therefore \quad L = \frac{v}{4\nu} = \frac{330}{4 \times 264} = 31.25 \text{ cm}$$

For first overtone,

Resonance length $= 3L = 93.75$ cm

For second overtone,

Resonance length $= 5L = 156.25$ cm

Options (*a*) and (*c*) are correct

38. (*a*) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L} = 512 \text{ Hz}.$$

Fundamental frequency of open pipe,

$$\nu' = \frac{v}{4L} = 2\nu = 2 \times 512 = \mathbf{1024 \text{ Hz}}.$$

39. (*a*), (*d*) In case of second resonance, the same energy is shared by a larger number of particles as the air column is longer. Hence energy per particle is less resulting in lower intensity. Option (*a*) is correct.

In first resonance,

$$l+e=\frac{\lambda}{4},\quad e=\text{end correction}$$

Due to end correction, l is slightly less than $\lambda/4$. Option (*d*) is correct.

The prongs of the tuning fork are kept in a vertical plane above the resonance tube.

40. (*d*) First harmonic of closed pipe

= Third harmonic of open pipe

$$\frac{v}{4L_1}=3\times\frac{v}{2L_2}$$

$$\therefore\quad \frac{L_1}{L_2}=\frac{1}{6}.$$

41. (*a*), (*b*), (*d*) For a closed pipe,

$$\nu=n\frac{v}{4L},\quad n=1,3,5,\ldots$$

$$\nu_1=\frac{v}{4L}=\frac{320}{4\times1}=\mathbf{80\ Hz}.$$

$$\nu_3=3\nu_1=\mathbf{240\ Hz}$$

$$\nu_5=5\nu_1=\mathbf{400\ Hz}.$$

42. (*b*) Mass per unit length of the string,

$$m=\frac{10^{-2}}{0.4}=2.5\times10^{-2}\ \text{kg m}^{-1}$$

Velocity of the wave in the string,

$$v=\sqrt{\frac{T}{m}}=\sqrt{\frac{1.6}{2.5\times10^{-2}}}=8\ \text{m/s}$$

For constructive interference between successive pulses, the minimum time interval is

$$\Delta t_{\min}=\frac{2L}{v}=\frac{2\times0.4}{8}=\mathbf{0.10\ s}.$$

43. (*b*), (*c*) $\nu=\frac{1}{2L}\sqrt{\frac{T}{m}}$

As $T_1>T_2$, so $\nu_1>\nu_2$

Initially, $\nu_1-\nu_2=6$ Hz.

When T_1 decreases, ν_1 decreases and it may be possible that $\nu_2-\nu_1$ becomes equal to 6 Hz. Hence option (*c*) is correct and option (*d*) is incorrect.

When T_2 increases, ν_2 increases and it may be possible that $\nu_2-\nu_1$ becomes equal to 6 Hz. Hence option (*b*) is correct and option (*a*) is incorrect.

44. (*b*) $\quad y=4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

$$\cos t=2\cos^2\left(\frac{t}{2}\right)-1$$

$$\Rightarrow\quad \cos^2\left(\frac{t}{2}\right)=\frac{1+\cos t}{2}$$

$$\therefore\quad y=2(1+\cos t)\sin(1000t)$$

$$=2\sin(1000t)+2\cos t\sin(1000t)$$

$$=2\sin(1000t)+\sin(1000t)+\sin(999t)$$

Clearly, y is the resultant of the superposition of three harmonic functions of angular frequencies 999, 1000 and 1001 rad/s.

45. (*a*), (*b*), (*c*), (*d*)

$$y(x,t)=0.02\cos\left(50\pi t+\frac{\pi}{2}\right)\cos(10\pi x)$$

Clearly, $a=0.02$ m

$$\omega=50\pi\ \text{rad s}^{-1},\quad k=10\pi\ \text{rad m}^{-1}$$

$$v=\frac{\omega}{k}=\frac{50\pi}{10\pi}=5\ \text{ms}^{-1}$$

∴ Option (*c*) is correct.

Displacement node occurs at

$$10\pi x=\frac{\pi}{2},\frac{3\pi}{2}\ \text{etc.}\quad\text{or}\quad x=\frac{1}{20},\frac{3}{20}$$

or $\quad x=0.05$ m and 0.15 m

∴ Option (*a*) is correct.

Displacement antinode occurs at

$$10\pi x=0,\pi,2\pi,3\pi,\ \text{etc.}$$

or $\quad x=0, 0.1$ m, 0.2 m, 0.3 m

∴ Option (*b*) is correct.

$\lambda=2\times$ distance between two consecutive nodes or antinodes.

$=2\times0.1=0.2$ m.

∴ Option (*d*) is correct.

46. (*a*), (*b*), (*c*) The number of waves encountered by the moving plane per unit time,

$$f'=\frac{\text{Distance travelled}}{\text{Wavelength}}=\frac{c+v}{\lambda}$$

$$=\frac{c}{\lambda}\left(1+\frac{v}{c}\right)=f\left(\frac{c+v}{c}\right)$$

∴ Option (*a*) is correct.

The stationary observer intercepts the incident wave of frequency f' and receives the reflected wave of frequency f'' emitted by the moving platform.

$$f''=\frac{f'}{1-v/c}=\frac{f(1+v/c)}{1-v/c}$$

$$=\frac{f(c+v)}{(c-v)}$$

∴ Option (*c*) is correct.

$$\lambda'' = \frac{c}{f''} = \frac{c(c-v)}{f(c+v)}$$

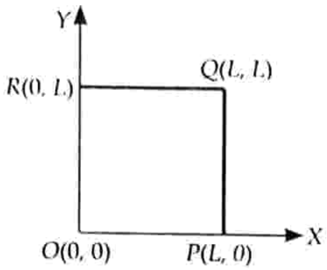
$\therefore$ Option (*b*) is correct.

Beat frequency $= f'' - f$

$$= \frac{f(1+v/c)}{(1-v/c)} - f = f\left(\frac{1+v/c}{1-v/c} - 1\right) = \frac{2vf}{c-v}$$

$\therefore$ Option (*d*) is incorrect.

47. (*b*), (*c*) As the edges of the square plate are clamped, displacement will be zero at the edges.



For option (*a*) :

$u(x, y) = 0$ at $x = L, y = L$

$u(x, y) \neq 0$ at $x = 0, \; y = 0$

For option (*b*) :

$u(x, y) = 0$ at $x = 0, y = 0$ $[\sin 0 = 0]$

$u(x, y) = 0$ at $x = L, \; y = L$ $[\sin \pi = 0]$

For option (*c*) :

$u(x, y) = 0$ at $y = 0, y = 0$ $[\sin 0 = 0]$

$u(x, y) = 0$ at $x = L, \; x = L\,[\sin \pi = 0, \; \sin 2\pi = 0]$

For option (*d*) :

$u(x, y) = 0$ at $y = 0, y = L$ $[\sin 0 = 0, \; \sin \pi = 0]$

$u(x, y) \neq 0$ at $x = 0, \; x = L\,[\cos 0 = 1, \; \cos 2\pi = 1]$

Hence only options (*b*) and (*c*) are correct.

48.

0	0	0	5

$$A = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\left(-\frac{\pi}{2}\right)} = 5$$

49.

0	0	0	7

Frequency of sound received by car, $f' = \left(\frac{v + v_c}{v}\right) f_0$

Frequency of sound reflected by the car is

$$f = \left(\frac{v}{v - v_c}\right) f' = \left(\frac{v + v_c}{v - v_c}\right) f_0$$

As $\quad v_c << v$, so $f = \left(\frac{1 + \frac{v_c}{v}}{1 - \frac{v_c}{v}}\right) f_0 = \left(1 - \frac{2v_c}{v}\right) f_0$

$$\therefore \quad \Delta f = \frac{-2 f_0}{v} \Delta v_c$$

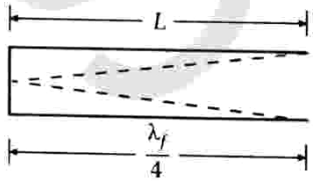
or $\quad \frac{1.2}{100} f_0 = \frac{-2 f_0}{v} \Delta v_c$

Difference, $\quad |\Delta v_c| = \frac{1.2}{100} \times \frac{300}{2}$ m / s

$$= \frac{1.2}{100} \times \frac{300}{2} \times \frac{18}{5} \approx \mathbf{7\ km/h}$$

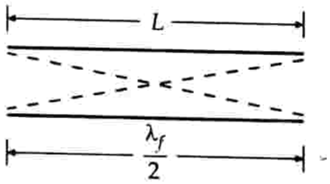
50. (*a*) $\to$ *p*, *t* ; (*b*) $\to$ *p*, *s* ; (*c*) $\to$ *q*, *s* ; (*d*) $\to$ *q*, *r*

(*a*) $\quad \frac{\lambda_f}{4} = L \Rightarrow \lambda_f = 4L$ (Longitudinal wave)



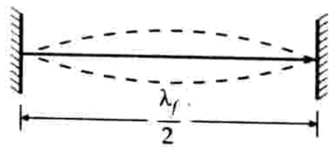
$\therefore$ (*a*) $\to$ *p*, *t* are the correct matching

(*b*) $\quad \frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$ (Longitudinal wave)



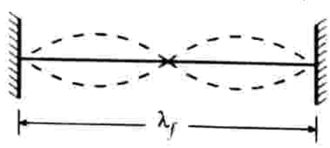
$\therefore$ (*b*) $\to$ *p*, *s* are the correct matching

(*c*) $\quad \frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$ (Transverse wave)



$\therefore$ (*c*) $\to$ *q*, *s* are the correct matching

(*d*) $\quad \frac{\lambda_f}{2} + \frac{\lambda_f}{2} = L \Rightarrow \lambda_f = L$ (Transverse wave)



$\therefore$ (*d*) $\to$ *q*, *r* are the correct matching.